Name

[illegible]

1. What is a ratio and what are the three ways to write a ratio?

____

____

2. What is a rate?

____

____

3. What is a unit rate? Give an example.

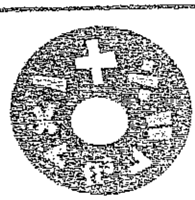
____

____

Ask Yourself

Can I simplify the numerator and the denominator?

Is this ratio a part-to-part ratio, a part-to-whole ratio, or a whole-to-part ratio?



Simplify each ratio, if possible. Then write it in two different ways.

4. 3 to 4 ______________

5. 4 to 3 ______________

6. $\frac{5}{20}$ ______________

7. $\frac{49}{42}$ ______________

8. 3:14 ______________

9. 22:4 ______________

17. There are 9 red gumdrops, 10 yellow gumdrops, and 15 orange gumdrops in a bag. What is the ratio of orange gumdrops to red gumdrops in the bag?

____

18. At a blood drive, 5 donors had type AB blood. The other 95 donors had other blood types. What was the ratio of donors with type AB blood to all donors?

____

19. Julia has a total of 12 T-shirts in her dresser. If 3 of the T-shirts are blue, what is the ratio of blue T-shirts to nonblue T-shirts in her dresser?

____

Independent Practice

1. What is a proportion?

____

____

2. What are equivalent ratios? Give an example of two equivalent ratios.

____

____

3. What are two different ways to prove that a proportion is true?

____

____

Solve each proportion.

20. If $\frac{1}{6} = \frac{x}{18}$, then $x =$ ____.

21. If $\frac{2}{7} = \frac{a}{49}$, then $a =$ ____.

22. If $\frac{3}{9} = \frac{12}{z}$, then $z =$ ____.

23. If $\frac{20}{24} = \frac{5}{x}$, then $x =$ ____.

24. If $\frac{6}{36} = \frac{m}{90}$, then $m =$ ____.

25. If $\frac{1}{5} = \frac{7}{n}$, then $n =$ ____.

26. If $\frac{x}{11} = \frac{3}{33}$, then $x =$ ____.

27. If $\frac{8}{z} = \frac{32}{48}$, then $z =$ ____.

Solve

28. The table below shows the ratio of paint to thinner that a painter will use for a particular job.

Paint (Number of Parts)	3	6	9	
Thinner (Number of Parts)	1	2	3	

If a mixture is 4 parts thinner, how many parts paint is it? Show your work, or explain how you determined your answer.

____

____

Proportion

Key Words
cross multiply
equivalent ratios
proportion

If two ratios have the same value when they are simplified, they are equivalent ratios. A proportion shows that two ratios are equivalent.

One way to prove that a proportion, $\frac{a}{b} = \frac{c}{d}$, is true is to **cross multiply**.

$$\frac{a}{b} = \frac{c}{d}$$

$ad = bc$ Equal cross products mean the proportion is true.

Another way to prove that a proportion is true is to use proportional reasoning. If you can multiply the numerator and denominator of one ratio by the same value to get the other ratio, the two ratios are equivalent.

You can use cross multiplication and proportional reasoning to solve for unknown values in proportions.

Example 1

Is $\frac{3}{5} = \frac{12}{20}$ a true proportion?

Use proportional reasoning.

What number times 3 equals 12? 4

So, multiply the numerator and denominator by 4.

$$\frac{3}{5} = \frac{3 \times 4}{5 \times 4} = \frac{12}{20} \checkmark$$

$\frac{3}{5} = \frac{12}{20}$ is a true proportion.

Example 2

Solve for x: $\frac{8}{12} = \frac{6}{x}$

Cross multiply.

$$\frac{8}{12} = \frac{6}{x}$$

$$8 \cdot x = 12 \cdot 6$$

$$8x = 72$$

$$x = 9$$

The solution is $x = 9$.

EXPLAIN

The $\stackrel{?}{=}$ symbol is used instead of the equal sign (=) when you are determining if a proportion is true or not. Explain why.

Guided Practice

Is $\frac{2}{12} = \frac{5}{28}$ a true proportion?

Step 1 Cross multiply.

$\frac{2}{12} \stackrel{?}{=} \frac{5}{28}$

$2 \cdot 28 \stackrel{?}{=} 12 \cdot$ _____

$56 \neq$ _____

Step 2 Is the proportion true?

_____, because the cross products are not __________.

The proportion __________ true, because $\frac{2}{12}$ and $\frac{5}{28}$ __________ equivalent ratios.

REMEMBER
If a proportion is true, the cross products will be equal.

The values in the table represent equivalent ratios. Find another equivalent ratio for this table.

First Term	2	3	4	?
Second Term	6	9	12	?

Step 1 Determine how the numbers in each row are related.

The numbers in the first row are increasing by 1.

The numbers in the second row are increasing by _____.

THINK
6 plus what equals 9?
9 plus what equals 12?

Step 2 Use those patterns to find a fourth equivalent ratio.

+ 1 + 1

First Term	2	3	4	
Second Term	6	9	12	

+ 3 + 3

REMEMBER
You can keep extending a table to find many equivalent ratios.

_____ is an equivalent ratio, because $\frac{2}{6} = \frac{3}{9} = \frac{4}{12} =$ _____.

[illegible]

1. What is a proportion?

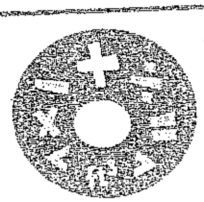
2. What are equivalent ratios? Give an example of two equivalent ratios.

3. What are two different ways to prove that a proportion is true?

Ask Yourself

For each set of ratios, which method will be faster or easier?

How can I use the numbers in the table to find an equivalent ratio?



Determine if each pair of ratios is equivalent or not. Then write the correct symbol (= or ≠) in the blank.

4. $\frac{1}{8}$ ___ $\frac{2}{16}$

5. $\frac{1}{3}$ ___ $\frac{4}{7}$

6. $\frac{2}{5}$ ___ $\frac{16}{40}$

7. $\frac{3}{4}$ ___ $\frac{15}{25}$

8. $\frac{4}{7}$ ___ $\frac{20}{35}$

9. $\frac{9}{10}$ ___ $\frac{32}{40}$

Solve.

10. The table shows three equivalent ratios. Use the table to find a fourth equivalent ratio. In the space below, use cross multiplication to show that your ratio is equivalent to one of the other ratios in the table.

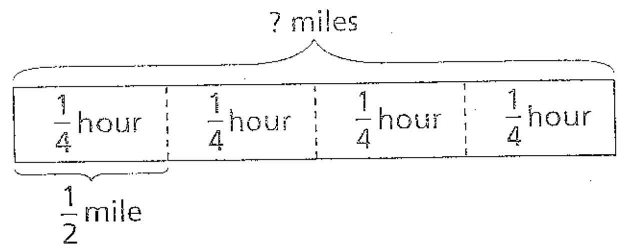
First Term	2	3	4	
Second Term	16	24	32	

Essential question: *How do you find and compare unit rates?*

CC.7.RP.1
CC.7.NS.3

Jeff hikes $\frac{1}{2}$ mile every $\frac{1}{4}$ hour. Lisa hikes $\frac{1}{3}$ mile every $\frac{1}{6}$ hour. How far do they each hike in 1 hour? 2 hours?

Use the bar diagram to help you determine how many miles Jeff hikes. How many $\frac{1}{4}$ hours are in 1 hour? How far does Jeff hike in 1 hour?

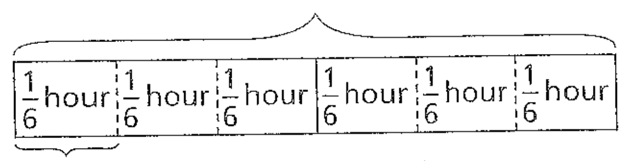


____

B Complete the table for Jeff's hike.

Distance (mi)	$\frac{1}{2}$				
Time (h)	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{3}{4}$	1	2

C Complete the bar diagram to help you determine how far Lisa hikes. How many miles does she hike in one hour?



____

D Complete the table for Lisa's hike.

Distance (mi)	$\frac{1}{3}$				
Time (h)	$\frac{1}{6}$	$\frac{1}{3}$	$\frac{1}{2}$	1	2

REFLECT

1a. How did you find Jeff's distance for $\frac{3}{4}$ hour?

____

1b. Which hiker walks farther in one hour? What does this tell you about the speeds at which they each hike?

____

____

A ratio is used to compare two quantities. When these quantities have different units, the ratio is called a rate. Ratios and rates can be expressed as fractions. When a rate has a denominator of 1, it is called a **unit rate**. To find a unit rate, divide the numerator by the denominator.

Sometimes rates are expressed as complex fractions. A **complex fraction** is a fraction that has a fraction as its numerator, denominator, or both. To simplify a complex fraction, use what you know about dividing rational numbers and divide the fraction in the numerator by the fraction in the denominator.

$$\frac{\frac{a}{b}}{\frac{c}{d}} = \frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c}$$

2 EXAMPLE Finding Unit Rates

While remodeling his kitchen, Arthur paints the cabinets. He estimates that he paints 30 square feet every half-hour. How many square feet does Arthur paint per hour?

Step 1: Find Arthur's rate for painting the cabinets.

$\frac{\_\_\_\_ \text{ square feet}}{\_\_\_\_ \text{ hour}}$

Step 2: Find the unit rate.

$\frac{30}{\frac{1}{2}} = 30 \div$ _Rewrite the fraction._

$= \frac{30}{1} \times$ _To divide, multiply by the reciprocal._

$=$ ________ _Multiply to find the unit rate._

Arthur paints ________________________________.

TRY THIS!

2a. Paige mows $\frac{1}{6}$ acre in $\frac{1}{4}$ hour. How many acres does Paige mow per hour?

____

____

REFLECT

2b. How could you find the unit rate for **2a** by using a table? Complete the table.

Acres	$\frac{1}{6}$			
Time (h)	$\frac{1}{4}$			

____

____

Scrambled Ratio and Proportions Name: ____________

6. Kevin runs 3 miles in 24 minutes. At the same rate, how long does it take him to run 1 mile?

Answer: __________

6. Jerry got 2 hits for every 7 times at bat. Jerry was at bat 56 times. How many hits did he get?

Answer: __________

3. Solve for n.

$$\frac{5}{2} = \frac{60}{n}$$

Answer: __________

7. Max was paid \$12.25 for working 3 hours. At this rate, how much did he earn in 6 hours?

Answer: __________

9. Two boxes of cereal cost \$1.74. How much will 5 boxes cost?

Answer: __________

17. Six tickets cost \$42. How much will 7 tickets cost?

Answer: __________

10. Curt earns \$23 in one week. At the same rate, how many weeks will it take him to earn \$161?

Answer: __________

5. Solve for n.

$$\frac{27}{81} = \frac{33}{n}$$

Answer: __________

4. On Lou's map, $\frac{1}{2}$ inch = 60 miles. The map distance between Denver and Pueblo is 1 inch. What is the actual distance?

Answer: __________

20. Stu saved \$3, which was 20% of the total price of the book. What was the total price of the book?

Answer: __________

Proportional Relationships, Tables, and Equations

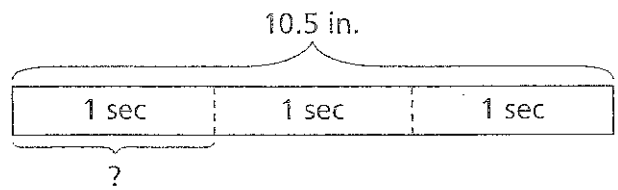
CC.7.RP.2a
CC.7.RP.2b
CC.7.RP.2c
CC.7.RP.3

Essential question: *How can you use tables and equations to identify and describe proportional relationships?*

1 EXPLORE Discovering Proportional Relationships

A giant tortoise moves at a slow but steady pace. It takes the giant tortoise 3 seconds to travel 10.5 inches.

A Use the bar diagram to help you determine how many inches a tortoise travels in 1 second. What operation did you use to find the answer?



____

B Complete the table.

Time (sec)	1	2	3	4	5
Distance (in.)			10.5		

C For each column of the table, find the ratio of the distance to the time. Write each ratio in simplest form.

$\frac{\text{Distance}}{\text{Time}} = \_\_\_$ $\frac{\text{Distance}}{\text{Time}} = \_\_\_$ $\frac{\text{Distance}}{\text{Time}} = \_\_\_$ $\frac{\text{Distance}}{\text{Time}} = \_\_\_$ $\frac{\text{Distance}}{\text{Time}} = \_\_\_$

D What do you notice about the ratios? ______________________________

E **Conjecture** How do you think the distance a tortoise travels is related to the time?

____

REFLECT

1a. Suppose the tortoise travels for 12 seconds. Explain how you could find the distance the tortoise travels.

____

1b. How would you describe the rate or speed at which a tortoise travels?

____

A **proportional relationship** is a relationship between two quantities in which the ratio of one quantity to the other quantity is constant. A giant tortoise can live as long as 150 years. One reason these reptiles live so long is their slow heart rate. A giant tortoise's heart beats only 6 times per minute. The giant tortoise's heart rate is an example of a proportional relationship. The ratio of the number of heart beats to the number of minutes is 6.

2 EXAMPLE Identifying Proportional Relationships

Alberto types 45 words per minute. Is the relationship between the number of words and the number of minutes a proportional relationship? Why or why not?

A Complete the table.

Time (min)	1	2	3	4	5
Number of Words	45				

B Complete the ratios.

$\frac{\text{Number of Words}}{\text{Time}} = \frac{45}{1} = \_\_\_ = \_\_\_ = \_\_\_ = \_\_\_ =$

The ratios are ______________________.

The *common ratio* is ______________________.

So, the relationship is ______________________.

TRY THIS!

2a. The table shows the distance Allison drove on one day of her vacation. Is the relationship between the distance and the time a proportional relationship? Why or why not?

Time (h)	1	2	3	4	5
Distance (mi)	65	120	195	220	300

____

REFLECT

2b. Do you think Allison drove at a constant speed for the entire trip? Why or why not?

____

The equation for a proportional relationship has a special form. If the relationship between x and y is a proportional relationship, then the equation for the relationship may be written as $y = ax$, where a is a positive number. The constant a is called the **constant of proportionality**.

$y = ax$

constant of proportionality

EXAMPLE 3 Writing an Equation for a Proportional Relationship

Two pounds of cashews cost $5, 3 pounds of cashews cost $7.50, and 8 pounds of cashews cost $20. Show that the relationship between the number of pounds of cashews and the cost is a proportional relationship. Then write an equation for the relationship.

Make a table to find the common ratio. Then write an equation with the common ratio as the constant of proportionality.

A Complete the table.

Number of Pounds	2	3	8
Cost ($)	5		

B Complete the ratios. $\frac{\text{Cost}}{\text{Number of Pounds}} = \frac{5}{2} = \underline{\quad} = \underline{\quad} =$

The common ratio is ______________.

C To write an equation, first tell what the variables represent.

Let x represent the number of pounds of cashews.

Let y represent the cost in dollars.

Use the common ratio as the constant of proportionality.

So, the equation for the relationship is ______________.

REFLECT

3a. How can you use substitution to check your equation?

____

3b. What is the unit cost (unit rate) for the cashews? How does the unit cost appear in your equation?

____

3c. How can you use your equation to find the cost of 6 pounds of cashews?

____

Tell whether the relationship is a proportional relationship. If so, give the constant of proportionality.

1.

Number of Minutes	3	4	5	6	7
Number of Seconds	180	240	300	360	420

2.

Time (h)	1	2	3	4	5
Biking Distance (mi)	12	26	36	44	50

3. Naomi reads 9 pages in 27 minutes, 12 pages in 36 minutes, 15 pages in 45 minutes, and 50 pages in 150 minutes.

4. A scuba diver descends at a constant rate of 8 feet per minute.

Write an equation for the relationship. Tell what the variables represent.

5. It takes Li 1 hour to drive 65 miles, 2 hours to drive 130 miles, and 3 hours to drive 195 miles.

6. There are 3.9 milligrams of calcium in each ounce of cooked chicken.

7.

Gallons of Gasoline	3	4	5	6
Total Cost ($)	9.45	12.60	15.75	18.90

8.

Cups of Batter	2	6	8	12
Number of Muffins	5	15	20	30

Information on three car rental companies is given.

9. Write an equation that gives the cost y of renting a car for x days from Rent-All.

10. What is the cost per day of renting a car from A-1?

11. Which company offers the best deal? Why?

Rent-All

Days	3	4	5	6
Total Cost ($)	55.50	74.00	92.50	111.00

A-1 Rentals
The cost y of renting a car for x days is given by $y = 22.5x$.

Car Town
The cost of renting a car from us is just $19.25 per day!

Name ______________________ Class ______________ Date __________

COMMON CORE

CC.7.RP.2a
CC.7.RP.2b
CC.7.RP.2d
CC.7.RP.3

Essential question: *How can you use graphs to represent and analyze proportional relationships?*

EXPLORE Graphing Proportional Relationships

Most showerheads that were manufactured before 1994 use 5 gallons of water per minute. Is the relationship between the number of gallons of water and the number of minutes a proportional relationship?

A Complete the table.

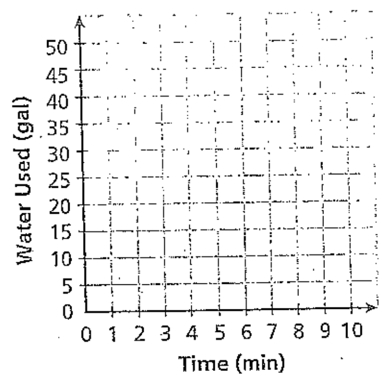
Time (min)	1	2	3		10
Water Used (gal)	5			35	

B Based on the table, is this a proportional relationship? Explain your answer.

____

C Plot the data from the table.

Showerhead Water Use



D **Draw Conclusions** If you continued the table to include 23 minutes, would the point (23, 125) be on this graph? Why or why not?

____

____

____

REFLECT

1a. **What If?** If a line was drawn through the plotted points, does it make sense that it would go through the origin? Explain.

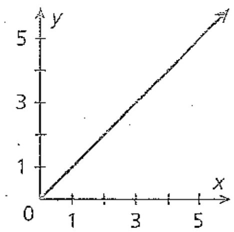
____

____

1b. Another showerhead uses less water per minute. How would its graph compare to the one you plotted?

____

In addition to using a table to determine if a relationship is proportional, you also can use a graph. A relationship is a proportional relationship if its graph is a straight line through the origin.



2 EXAMPLE Identifying Proportional Relationships

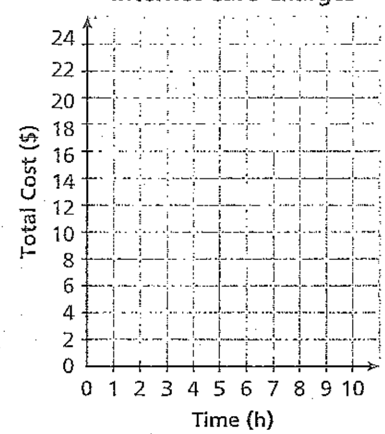
An Internet café charges a one-time $5 service fee and then $2 for every hour of use. Is this relationship a proportional relationship?

A Complete the table.

Time (h)	1	2	5		8
Total Cost ($)	7			17	

B Plot the data from the table and connect the points with a line.

Internet Café Charges



C The graph of the data is a ______________________.

The line does/does not go through the origin.

So, the relationship is ______________________.

TRY THIS!

2a. Plot the data from the table and connect the points with a line.

Canoe Rental (h)	2	5	8	10
Total Cost ($)	5	11	17	21

2b. Is this a proportional relationship? Explain.

____

____

____

Canoe Rental Fees