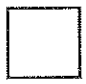
Properties of Polygons

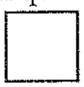
This activity is designed to reinforce the properties of polygons using a string tied to form a loop.

Work in groups of three. The first person should read the description of the polygon aloud. The other two people in the group will work together to manipulate the string to form the matching polygon and decided on the correct name. Each person should name the polygon and make a drawing of it to the right of the statement on his or her own paper. Switch roles so that each person has an opportunity to form polygons with the string. *Note:* Be sure to list all polygons for which the criteria hold.

1. An equilateral quadrilateral

   rhombus  square 

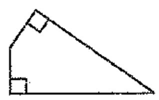
2. An equilateral quadrilateral with at least one right angle

   square 

3. An equilateral quadrilateral with exactly one right angle

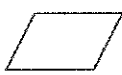
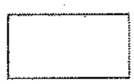
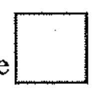
   None or DNE

4. A four-sided polygon with exactly two right angles

   Examples: trapezoid  kite 

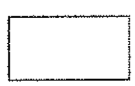
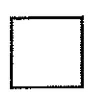
5. A polygon with at least two pairs of parallel sides

   Numerous possible answers: Full credit for ALL four quadrilaterals and at least 1 with 5 or more sides.

   parallelogram  rectangle  rhombus  square 

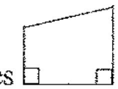
   hexagon  any regular n-gon, $n \geq 4$, n is even. Possible with a polygon with 5 or more sides and purposefully constructed

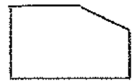
6. A quadrilateral with congruent diagonals

   isosceles trapezoid  rectangle  square 

7. A polygon with exactly one pair of parallel sides and at least two right angles

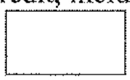
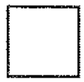
   At least 2 means 3 or more

   Half credit for trapezoid with exactly 2 right angles 

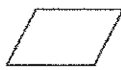
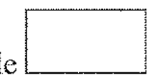
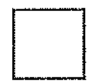
   Full credit for a purposefully constructed n-gon, n ≥ 5 

8. A polygon with at least one pair of parallel sides and at least two right angles

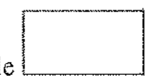
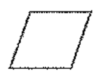
   For full credit, include at least one non-quadrilateral

   rectangle  square  any purposefully constructed n-gon, n $\geq$ 5 (see #7)

9. A quadrilateral with two pairs of congruent sides

   kite  parallelogram  rectangle  rhombus  square 

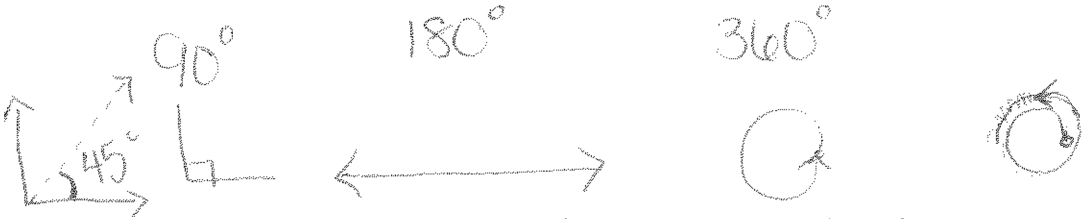
10. A quadrilateral with two pairs of congruent angles

    parallelogram  rectangle  rhombus 

    square  isos. trap. 

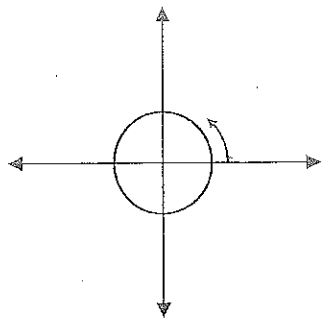
11. Choose a polygon and write a statement that describes the polygon. Read the description aloud. The rest of the group members should form the polygon with the string and name it correctly.

    ______Answers may vary.______________________________



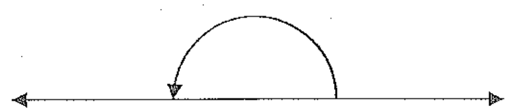
Estimating Angles

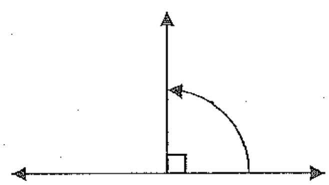
Degrees are the common units used to measure the size of angles.



Imagine starting at the positive x-axis and rotating a point around the rectangular plane until returning to the start position. The shape created is a circle, and the circle contains 360 degrees of rotation.

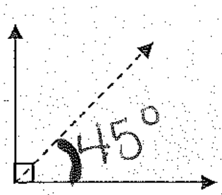
Half of the distance around the circle is a straight line. The measure in degrees for a straight line is 180°.



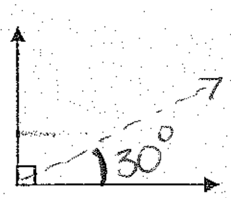


Half of the rotation of a straight line is the angle that connects perpendicular lines. This rotation is 90 degrees.

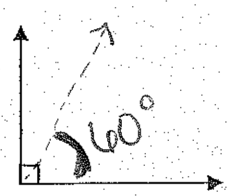
What would be the measure of rotation if the 90° angle were cut in half?



Draw an estimate of what a 30° angle and a 60° angle would look like:



30°



60°

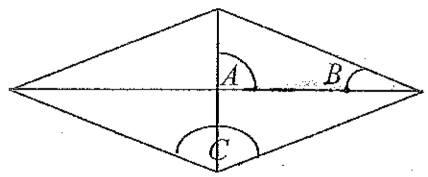
Name ______________________________

Practice with Angles

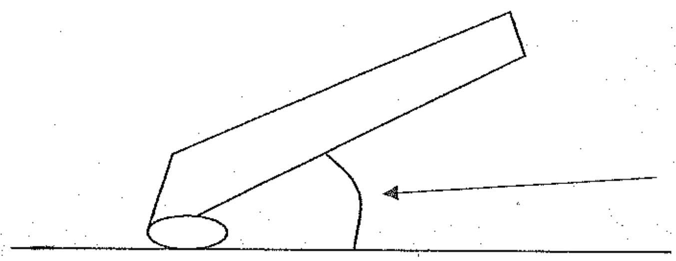
Directions: *Calculate or estimate the values for the angles in each question. Write the answers in the spaces provided.*

1 There are 24 time zones around the world. How many degrees does each time zone occupy?

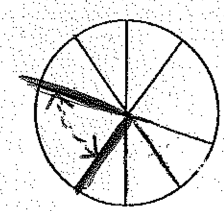
2 Label the angles indicated in the drawing as acute, obtuse, straight, or right.



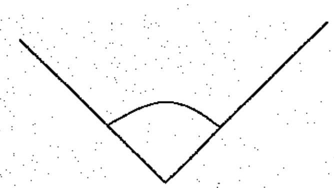
3 Estimate the angle at which the clown is being fired from the cannon.



4 Estimate the degree of the angles between the broken bicycle spokes (dotted lines) shown in the diagram below.



5 Estimate the measure in degrees of the angle shown.



3 Construct Geometric Shapes

Key Words

- acute triangle
- obtuse triangle
- right triangle
- triangle angle sum theorem
- triangle inequality theorem

One way to construct a geometric shape, such as a triangle, is to use computer drawing technology. Another way is to draw it by hand using a ruler, a protractor, or both. If you draw by hand, you can use trial and error to determine if it is possible to draw one unique triangle, more than one triangle, or no triangle given certain conditions. It is often helpful to try drawing different types of triangles, such as **acute triangles**, **obtuse triangles**, or **right triangles**.

You can also use several theorems to determine if a certain triangle is possible or not.

The **triangle angle sum theorem** states that the sum of the angles in any triangle is 180°.

The **triangle inequality theorem** states that the sum of the lengths of any two sides of a triangle is greater than the length of the third side.

Example

Is it possible to draw a triangle with sides measuring 2 centimeters, 3 centimeters, and 6 centimeters?

According to the triangle inequality theorem, if this triangle is possible, the sum of any two side lengths should be greater than the third side length.

$2 + 6 \overset{?}{>} 3$

$8 > 3$, so the inequality $2 + 6 > 3$ is true.

$3 + 6 \overset{?}{>} 2$

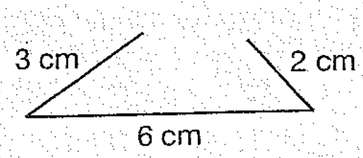
$9 > 2$, so the inequality $3 + 6 > 2$ is true.

$2 + 3 \overset{?}{>} 6$

$5 < 6$, so the inequality $2 + 3 > 6$ is not true.

This is not a possible triangle.

If you tried to construct it, you would not be able to connect all 3 sides, as shown:



It is impossible to draw a triangle with sides measuring 2 centimeters, 3 centimeters, and 6 centimeters.

CONSTRUCT

In the space below, construct a right triangle with two sides measuring 1 inch each. Is the third side longer than or equal to 1 inch in length?

Guided Practice

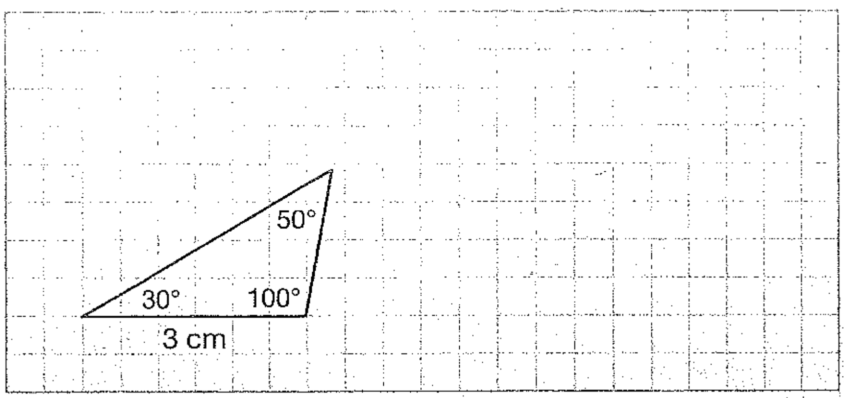
Is it possible to construct a triangle with angles measuring 30°, 50°, and 100°? If so, can you draw one unique triangle or many different triangles?

Step 1 Is the triangle possible?

30° + 50° + 100° = _______°, so the triangle _______ possible.

Step 2 Use a protractor to draw triangles with those angle measures.

One possible triangle is drawn for you. In the space below, draw a second triangle with the same angle measures but which is larger or smaller than the first triangle.



The triangle you drew is similar to the first triangle, because the corresponding angles are _________________________.

It _______ possible to draw many different triangles with angles measuring 30°, 50°, and 100°.

THINK

The triangle angle sum theorem states that the sum of the angles in a triangle is 180°.

REMEMBER

Similar triangles have the same shape but not necessarily the same size. So, a triangle with the given angle measures could be many different sizes.

Independent Practice

1. What does the triangle angle sum theorem state?

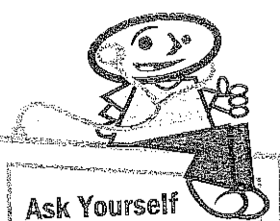
____

____

2. What does the triangle inequality theorem state?

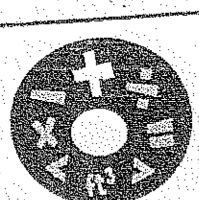
____

____



Ask Yourself

To test the triangle inequality theorem, should I add the lengths of the shortest two sides?



Is it possible to draw a triangle with the given angle measures? Write *yes* or *no*.

3. 20°, 70°, 90° ______

4. 45°, 45°, 100° ______

5. 32°, 56°, 82° ______

6. 16°, 34°, 130° ______

Is it possible to draw a triangle with the given side lengths? Write *yes* or *no*.

7. 5 m, 6 m, 12 m ______

8. 6 units, 7 units, 12 units ______

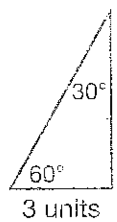
9. 12 ft, 15 ft, 22 ft ______

10. 1 in., 3 in., 4 in. ______

11. Is it possible to draw an obtuse triangle with side lengths of 2 centimeters, 2 centimeters, and 3 centimeters? Use a ruler to create a drawing below that shows that your answer is correct.

____

12. The grid below shows a triangle with angle measures 30°, 60°, and 90°.



Is this triangle unique, or is it possible to draw a different triangle with those same angle measures? Use a protractor to create a drawing on the grid above to illustrate your answer.

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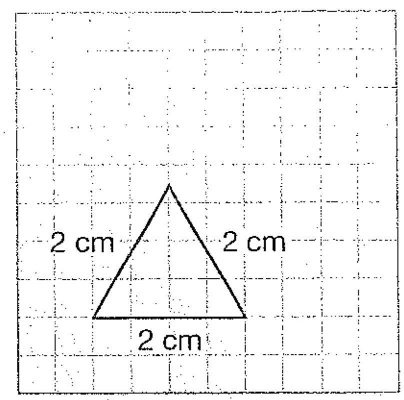
13. The grid at the right shows a triangle with sides measuring 2 centimeters, 2 centimeters, and 2 centimeters. Is this triangle unique, or is it possible to draw a different triangle with those same side lengths? Explain your answer.

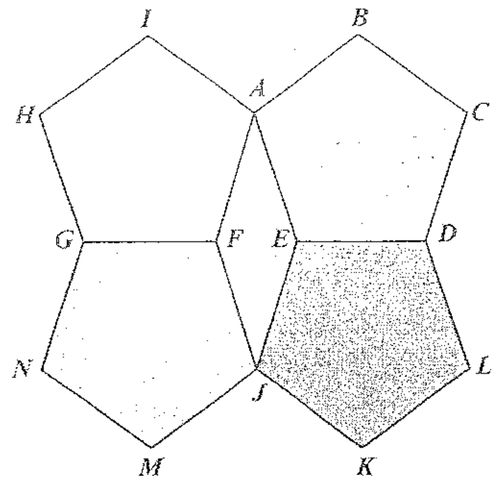


Solve.

14. Is it possible to draw a triangle with side lengths of $\frac{1}{2}$ inch, 1 inch, and 2 inches? Use the triangle inequality theorem and the lines below to explain your answer. Then create a drawing on the right to further show why your answer is correct.

Four Pentagons

This diagram is made up of four regular pentagons that are all the same size.



1. Find the measure of angle AEJ.

   Show your calculations and explain your reasons.

2. Find the measure of angle EJF.

   Explain your reasons and show how you figured it out.

3. Find the measure of angle KJM.

   Explain how you figured it out.

The Pentagon Problem

Mrs. Morgan wrote this problem on the board:

This pentagon has three equal sides at the top and two equal sides at the bottom.

Three of the angles have a measure of 130°.

Figure out the measure of the angles marked x and explain your reasoning.

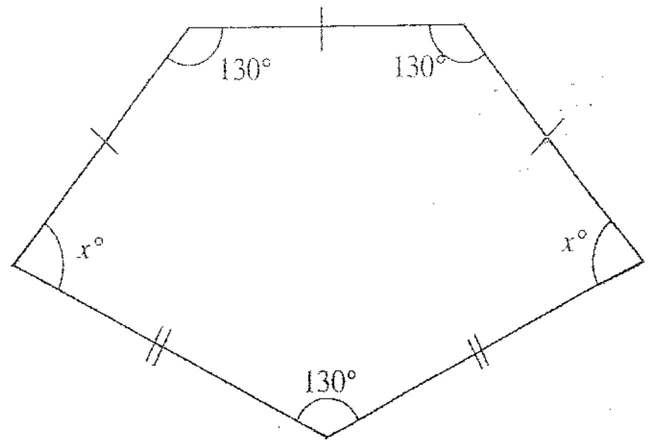


Diagram is not accurately drawn.

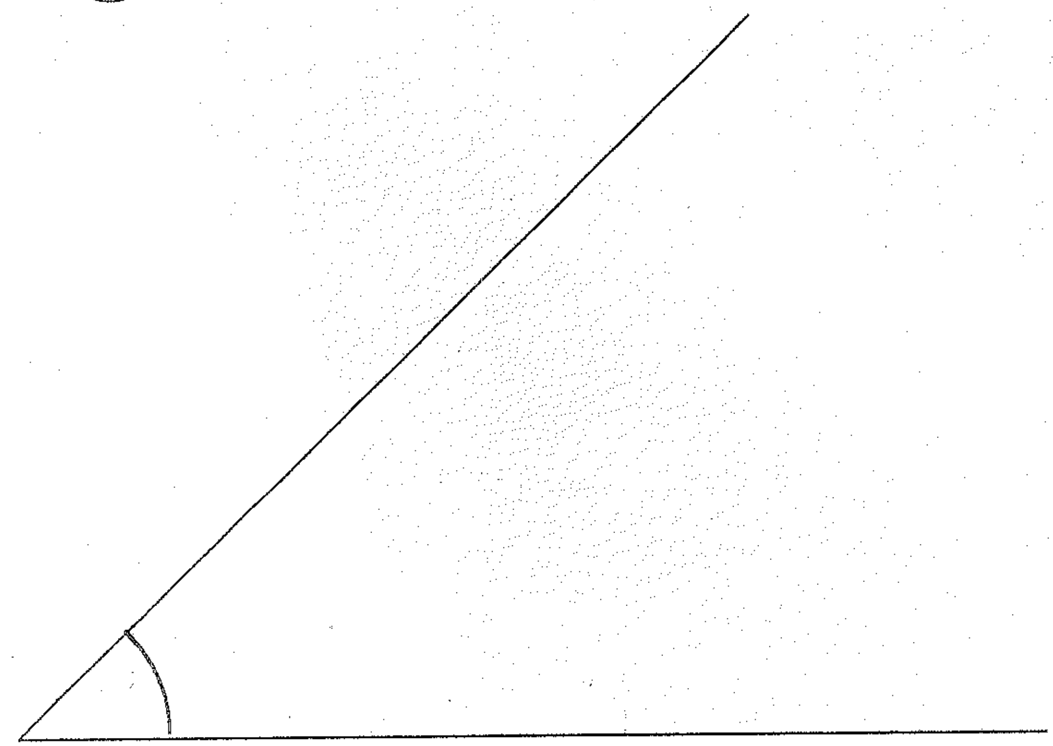
Four students in Mrs. Morgan's class came up with different methods for answering this problem.

Use each student's method to calculate the measure of angle x.

Write all your reasoning in detail.

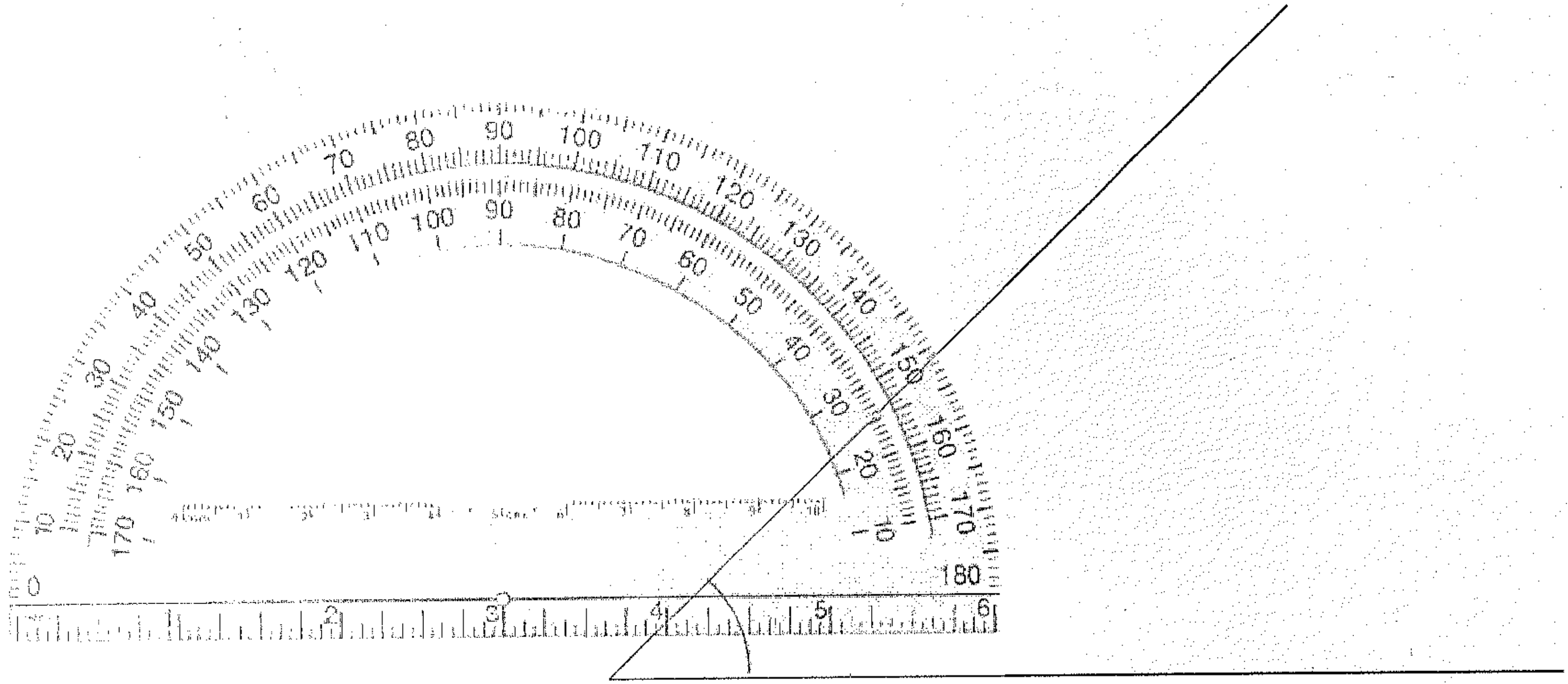
Use the *Geometrical Definitions and Properties* sheet to help.

Measuring an Angle



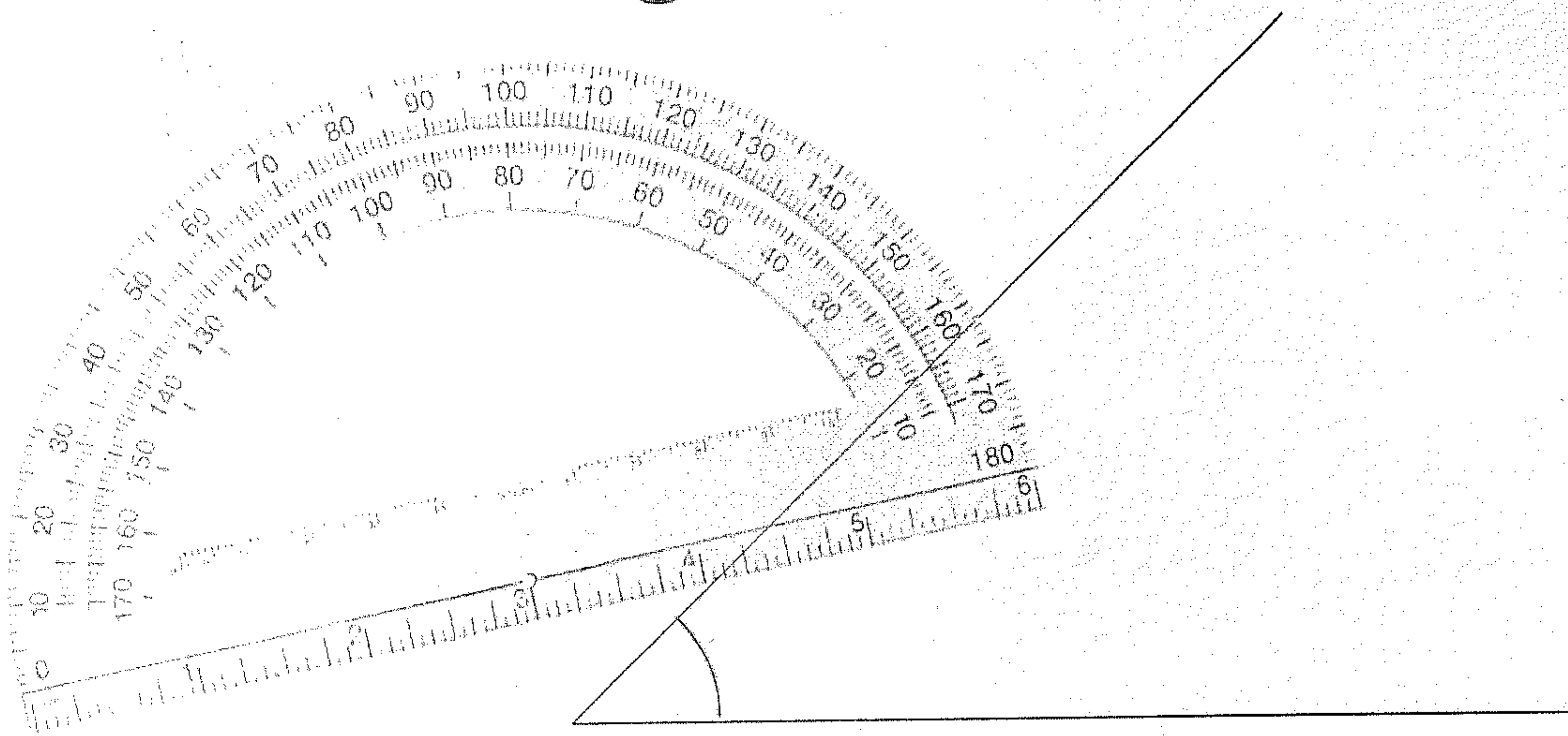
Step 1) Determine if the angle is acute or obtuse before using the protractor.

Measuring an Angle



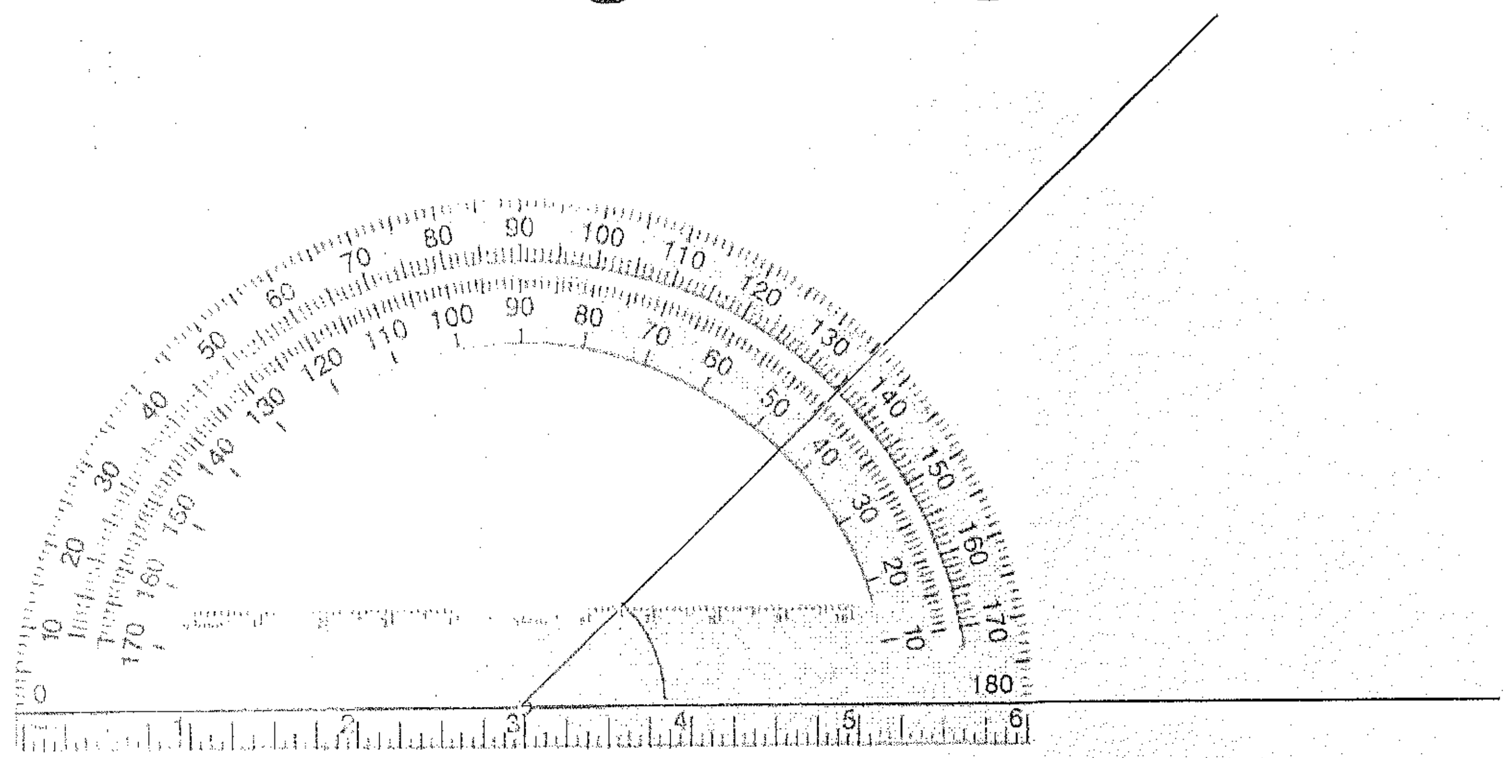
Step 2) Place the center mark of the protractor on the vertex of the angle

Measuring an Angle



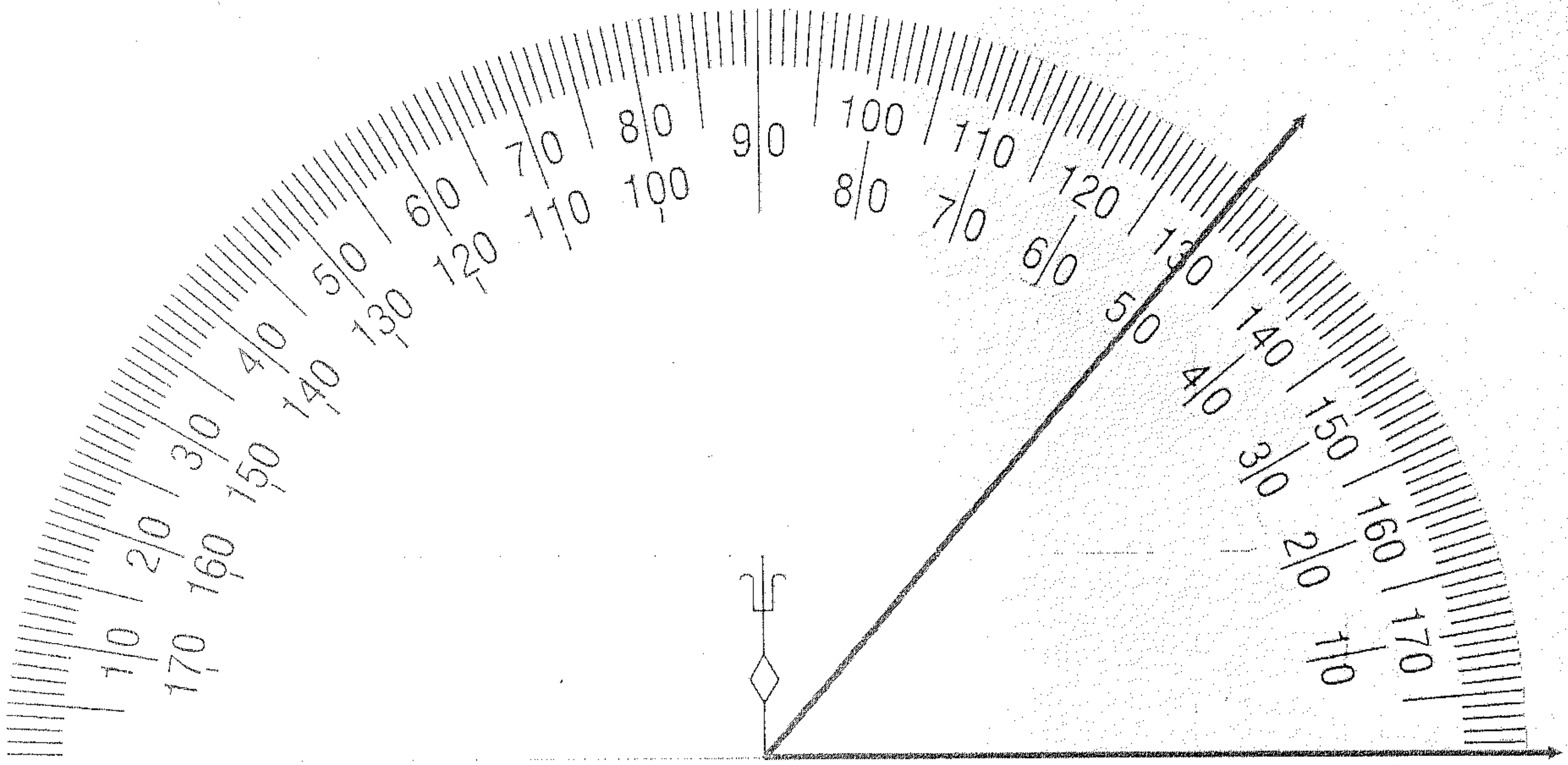
Step 3) Rotate the zero-edge of the protractor to line up with one ray of the angle and for the other ray of the angle to cross the protractor's scale.

Measuring an Angle

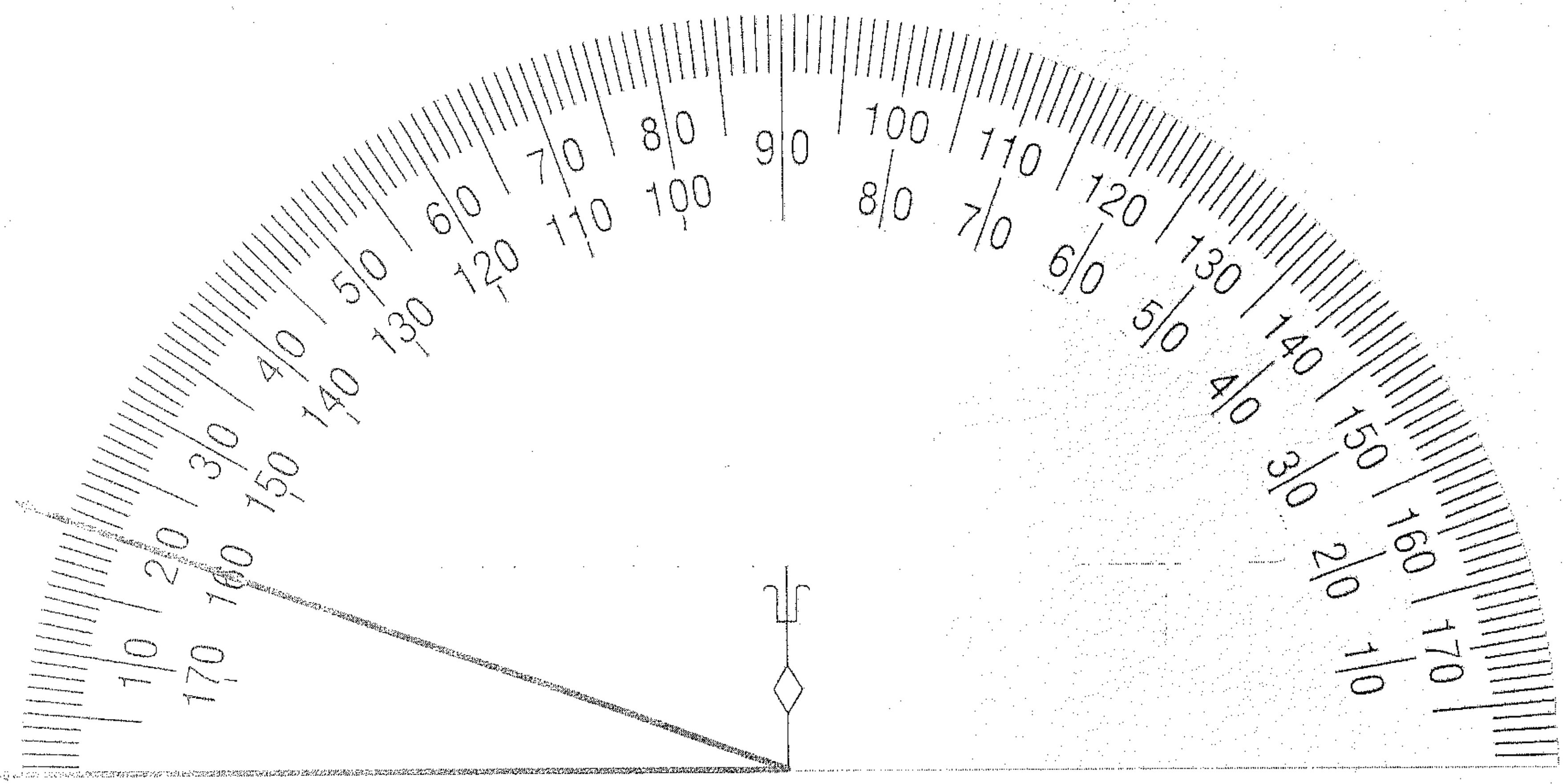


Step 4) Read the measure of the angle of the ray that crosses the protractor's scale.

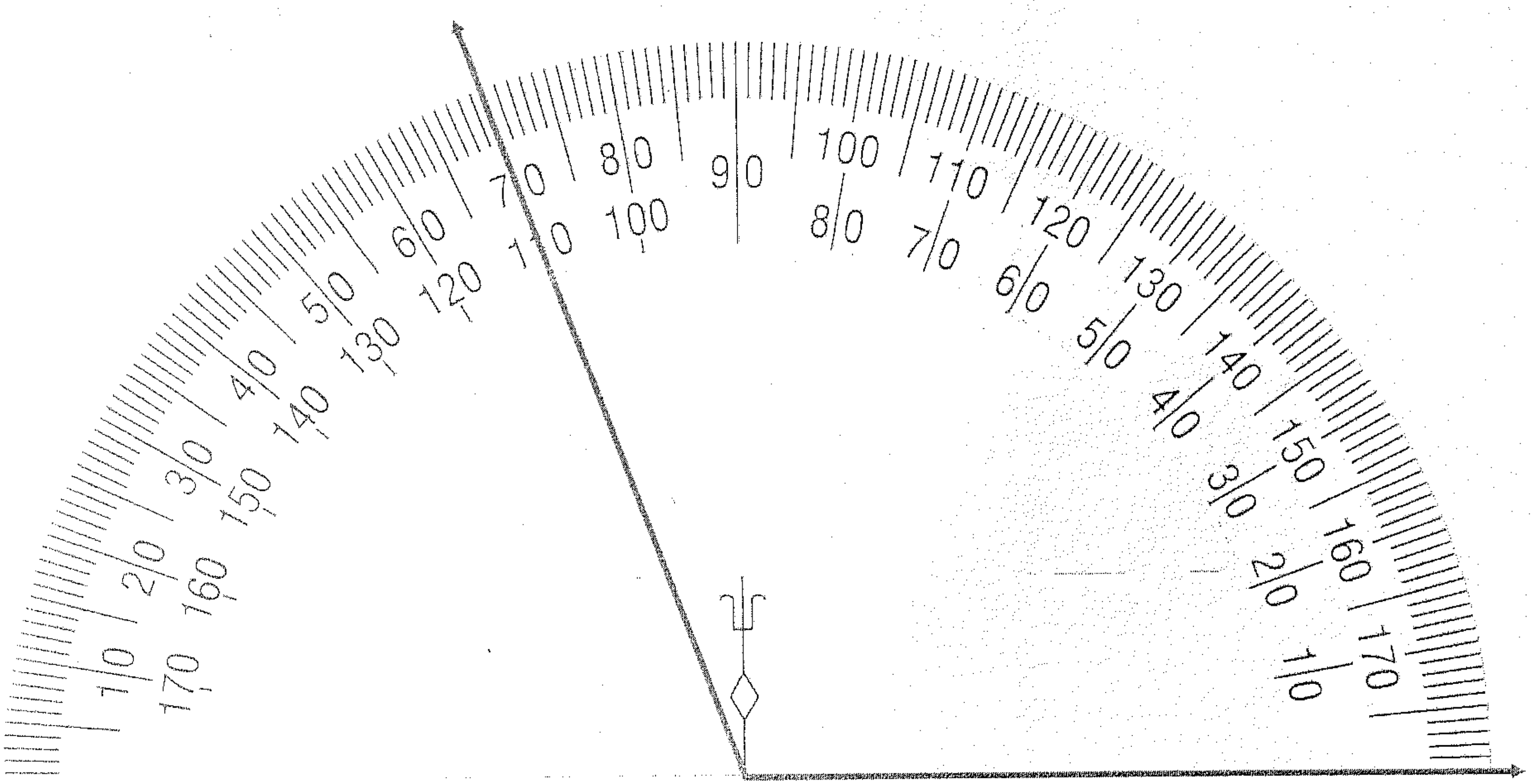
Is the measure of the angle 130 or 50 degrees? Explain.



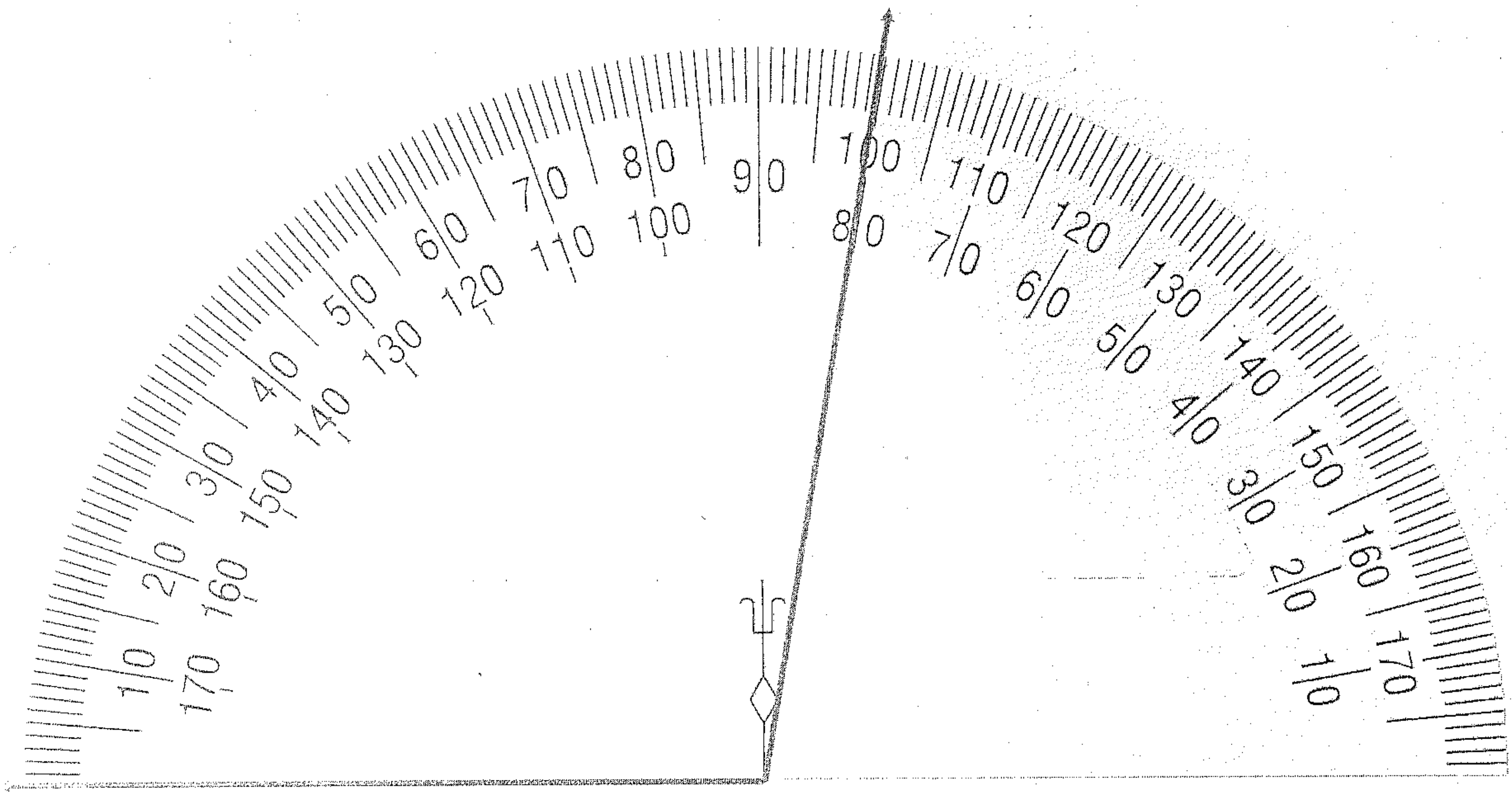
Is the measure of the angle 20 or 160 degrees? Explain.



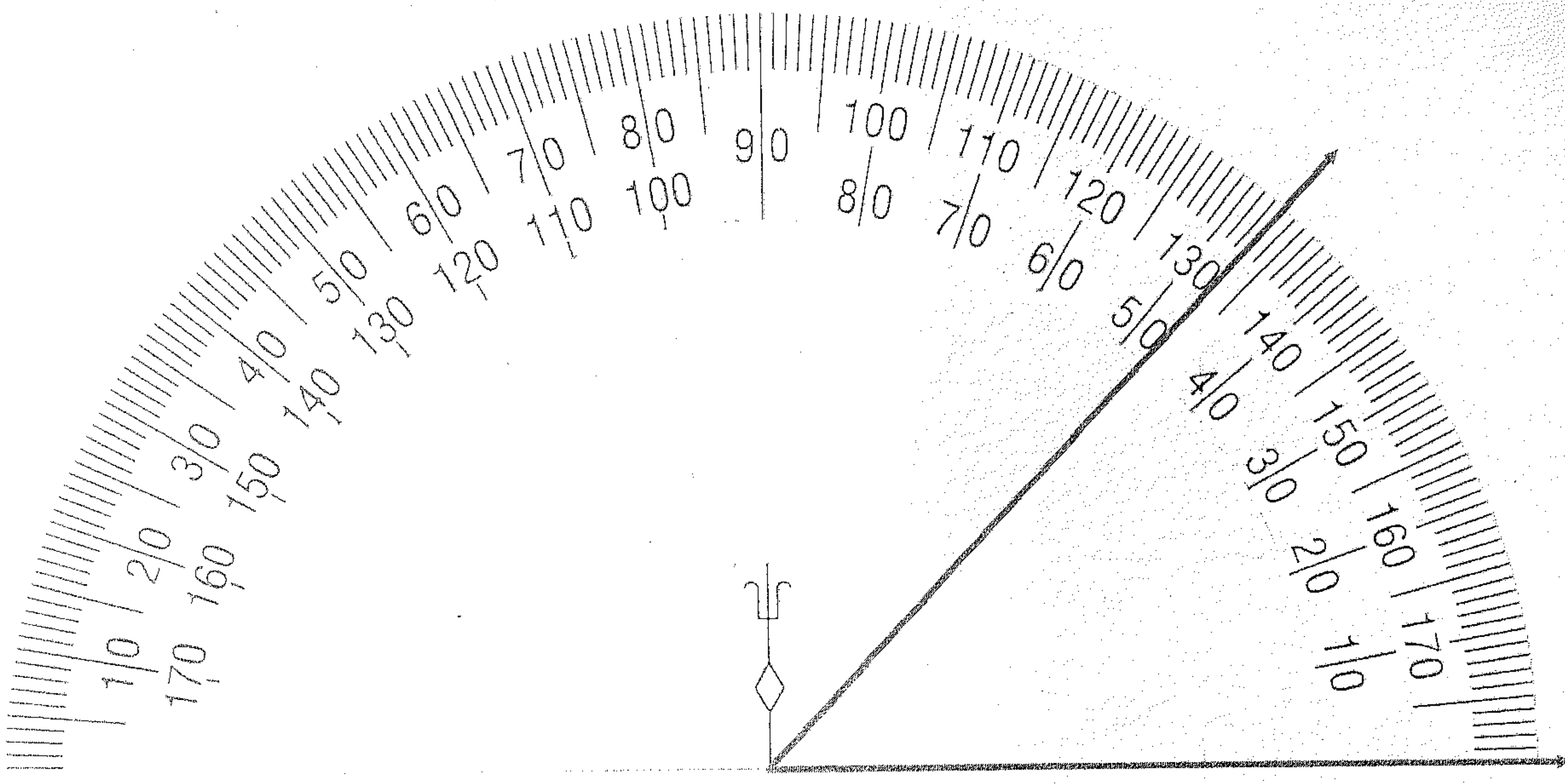
Is the measure of the angle 70 or 110 degrees? Explain.



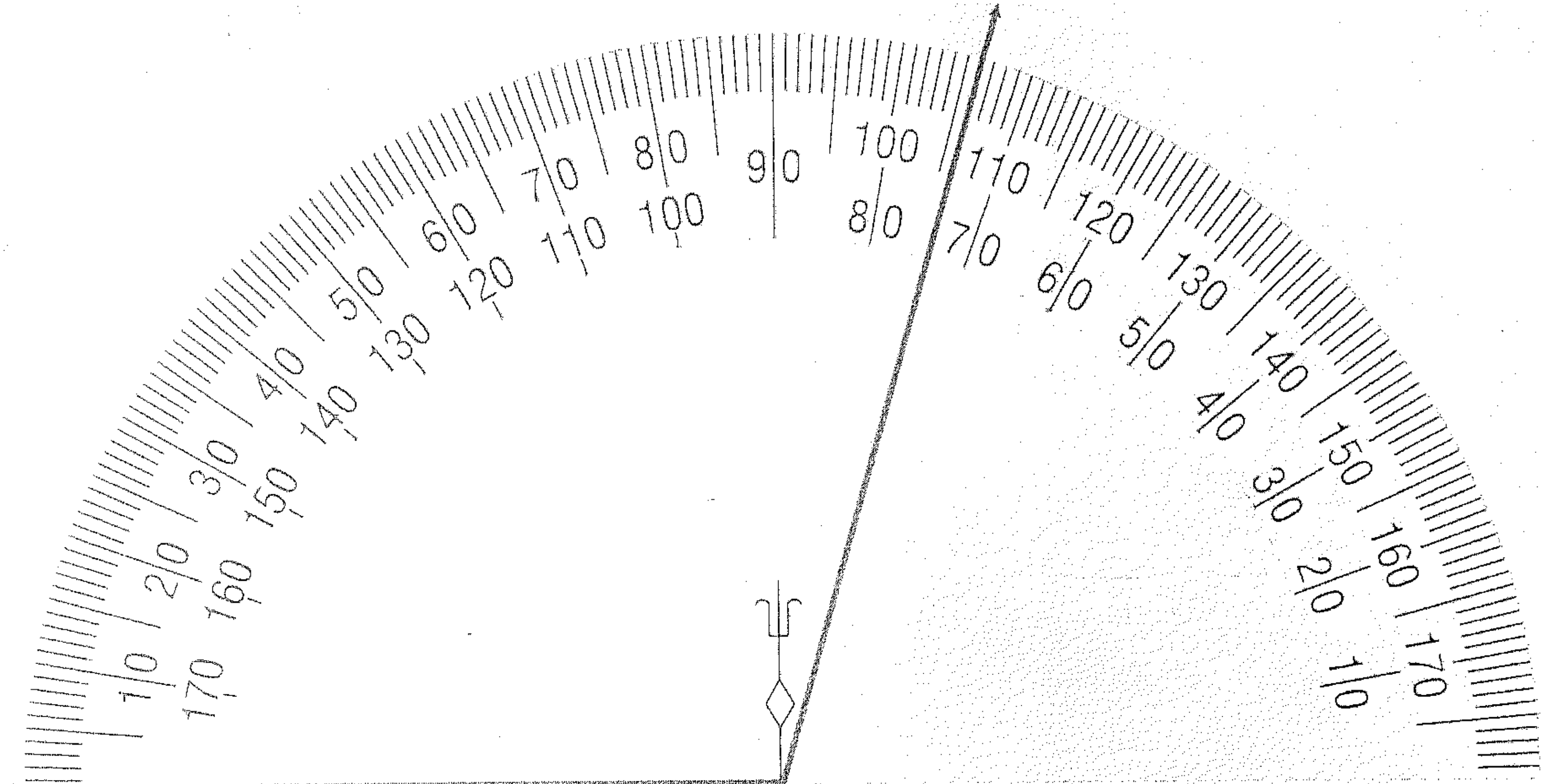
Is the measure of the angle 80 or 100 degrees? Explain.



Is the measure of the angle 53 or 47 degrees? Explain.



Is the measure of the angle 106 or 114 degrees? Explain.

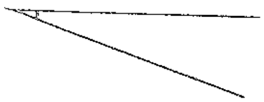


Measuring Angles

Name: ______________________ Date: ______________________

Use your protractor to extend the lines and measure each angle.

(1)

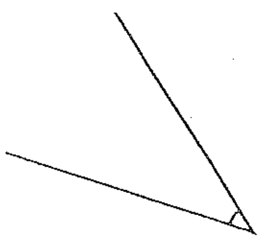


This angle is ____ degrees.

(2)

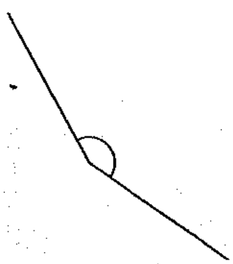
This angle is ____ degrees.

(3)



This angle is ____ degrees.

(4)

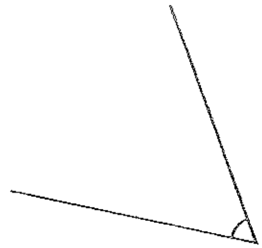


This angle is ____ degrees.

(5)

This angle is ____ degrees.

(6)

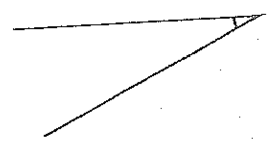


This angle is ____ degrees.

(7)

This angle is ____ degrees.

(8)



This angle is ____ degrees.

(9)

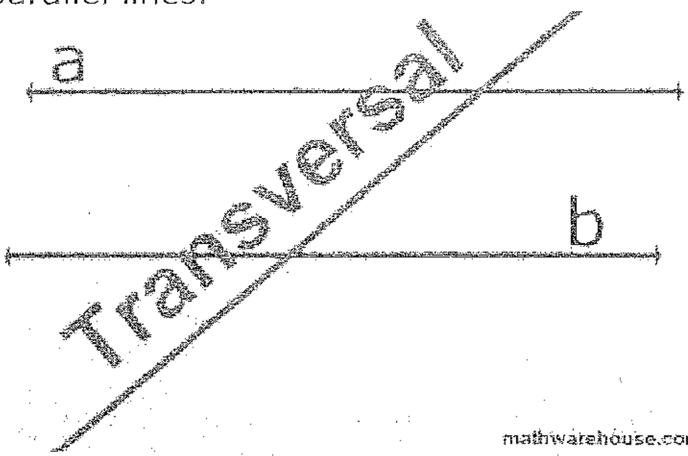
This angle is ____ degrees.

(10)

This angle is ____ degrees.

What is a transversal ?

Answer: A transversal is a line, like the red one below, that intersects to parallel lines.



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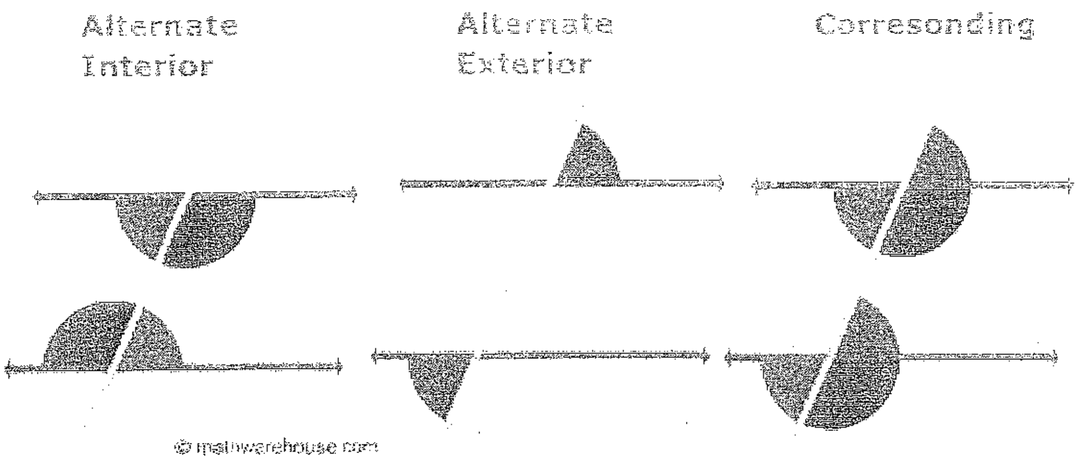
What is so special about a transversal ?

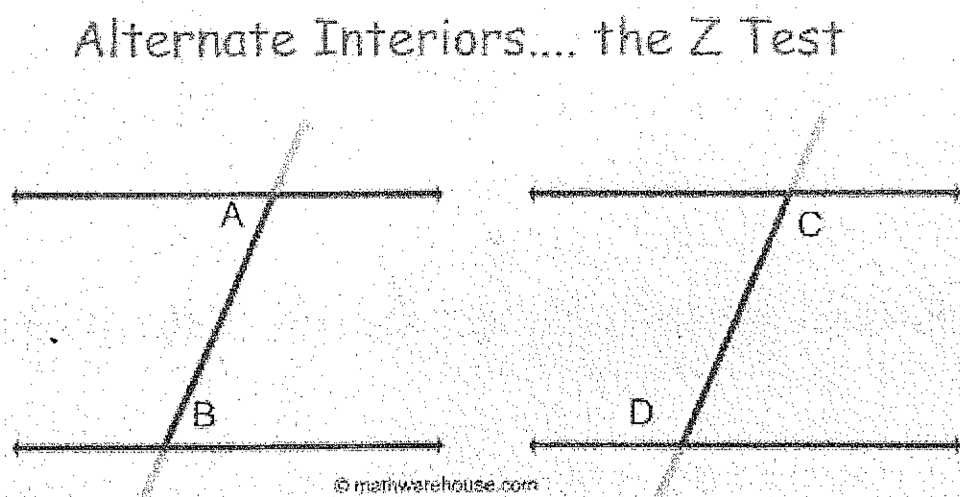
Answer: When a transversal cuts (or intersects) parallel lines several pairs of congruentand supplementary angles are formed.

The Congruent Angle Pairs

There are 3 types of angles that are congruent: Alternate Interior, Alternate Exterior and Corresponding Angles.

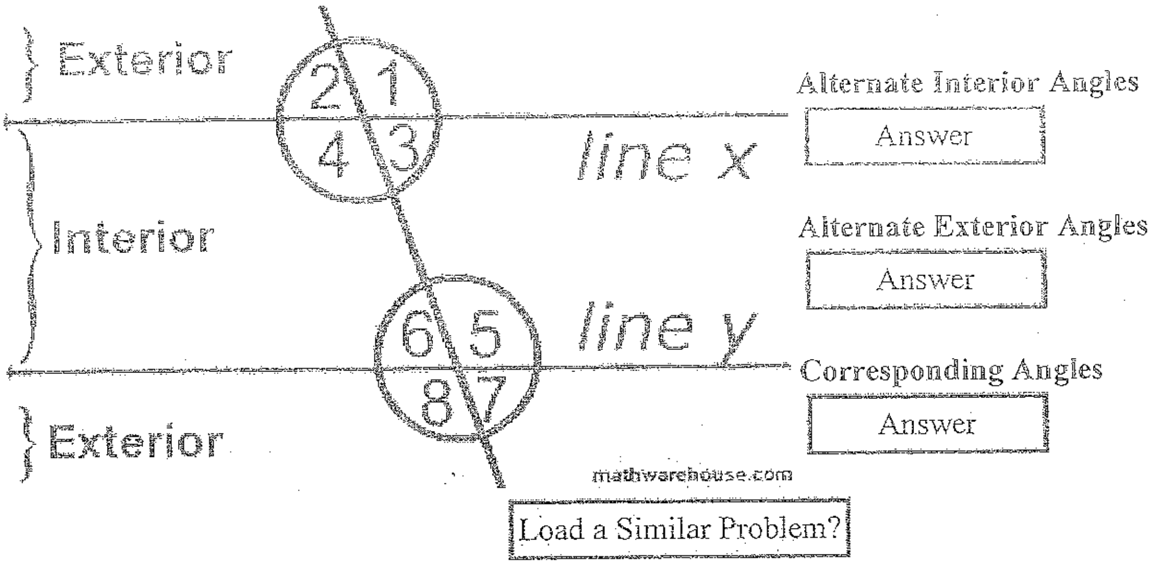


Can you make a Z?



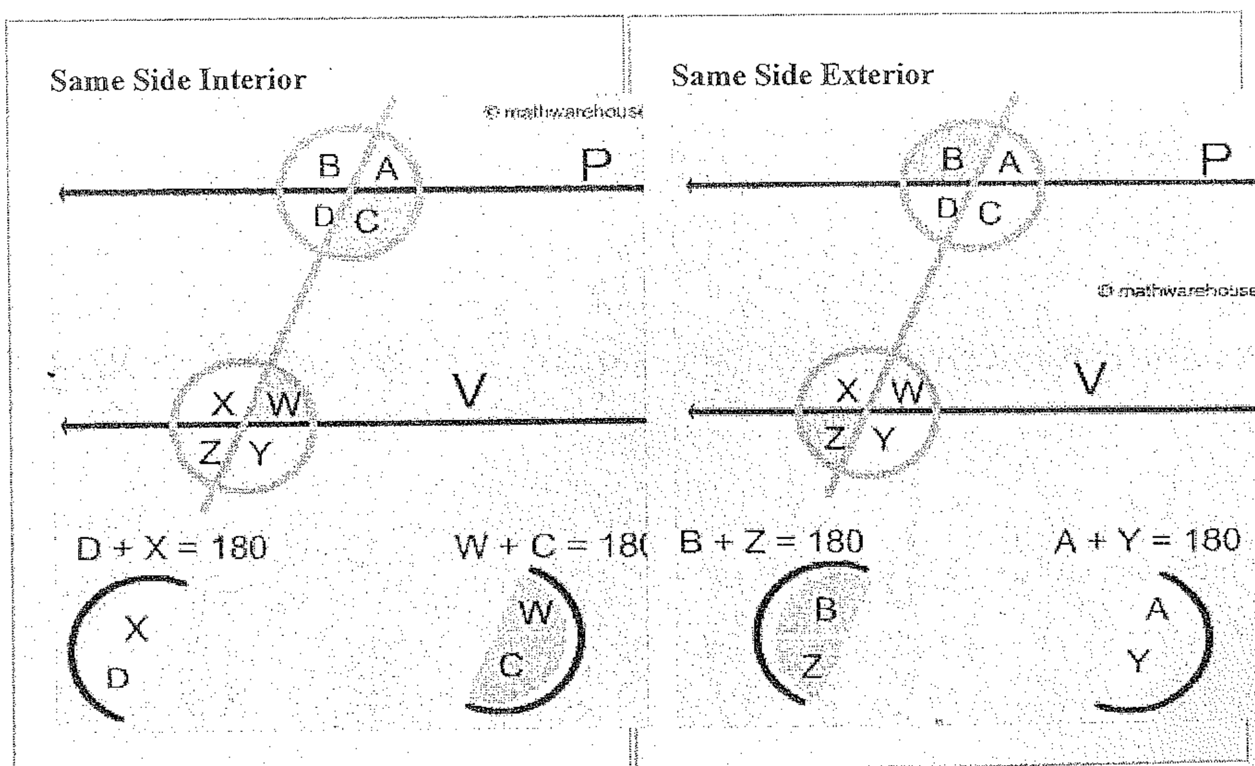
Some people find it helpful to use the 'Z test' for alternate interior angles.
If you can draw a Z or a 'backwards z', then the alternate interior angles are the ones that are in the corners of the *Z*

Can you identify each type of congruent angle in the picture below?



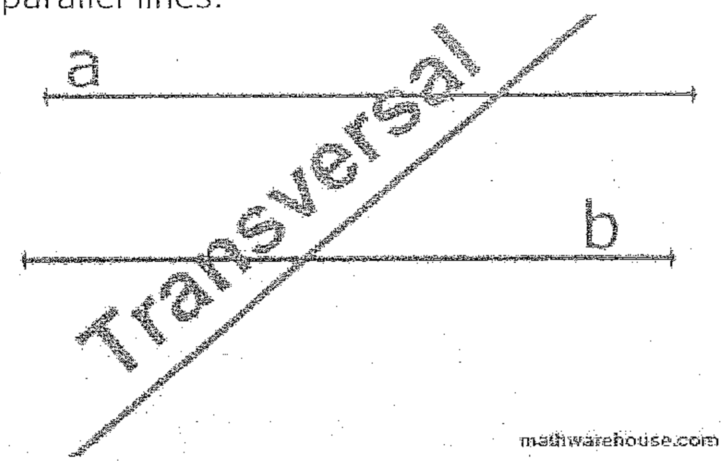
The Supplementary Angle Pairs

There are 2 types of supplementary angles that are formed: Same side interior and same side exterior



What is a transversal ?

Answer: A transversal is a line, like the red one below, that intersects to parallel lines.

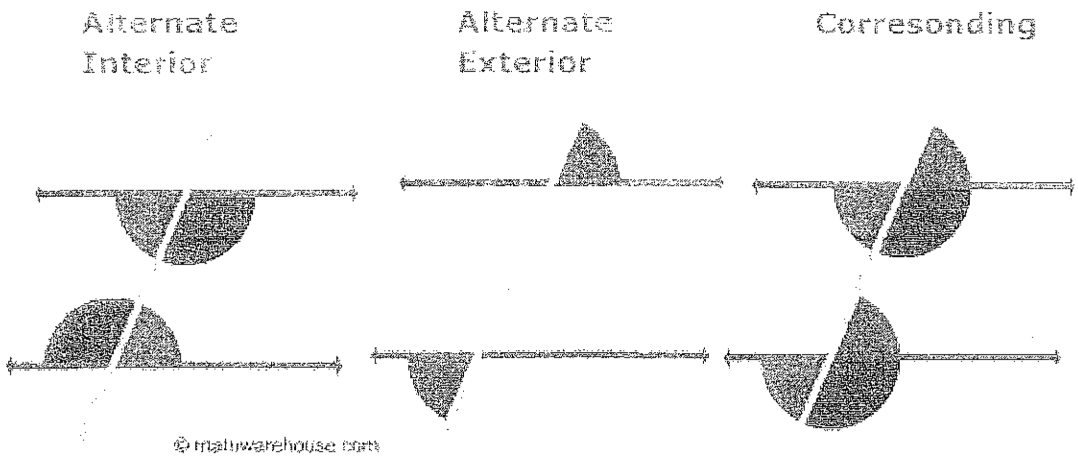


What is so special about a transversal ?

Answer: When a transversal cuts (or intersects) parallel lines several pairs of congruentand supplementary angles are formed.

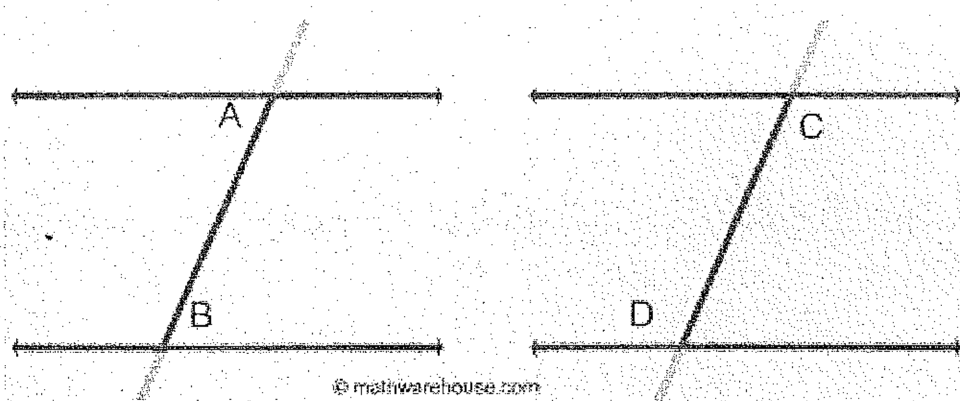
The Congruent Angle Pairs

There are 3 types of angles that are congruent: Alternate Interior, Alternate Exterior and Corresponding Angles.



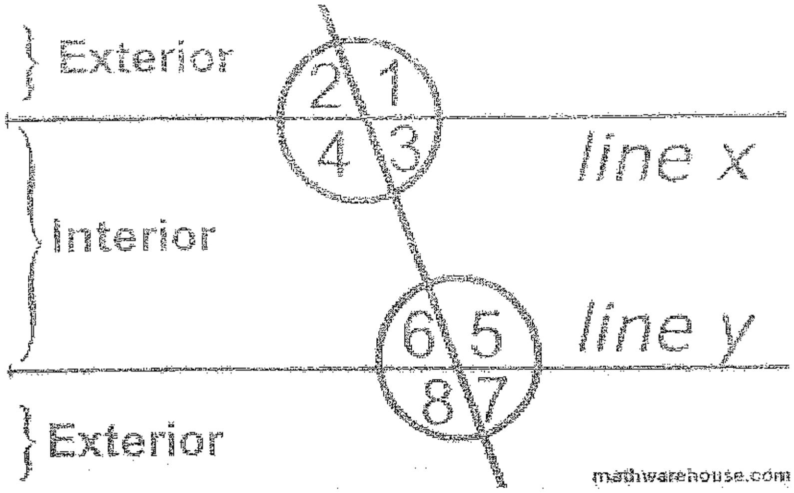
Can you make a Z?

Alternate Interiors.... the Z Test



Some people find it helpful to use the 'Z test ' for alternate interior angles. If you can draw a Z or a 'backwards z', then the alternate interior angles are the ones that are in the corners of the *Z*

Can you identify each type of congruent angle in the picture below?



Alternate Interior Angles

Answer

Alternate Exterior Angles

Answer

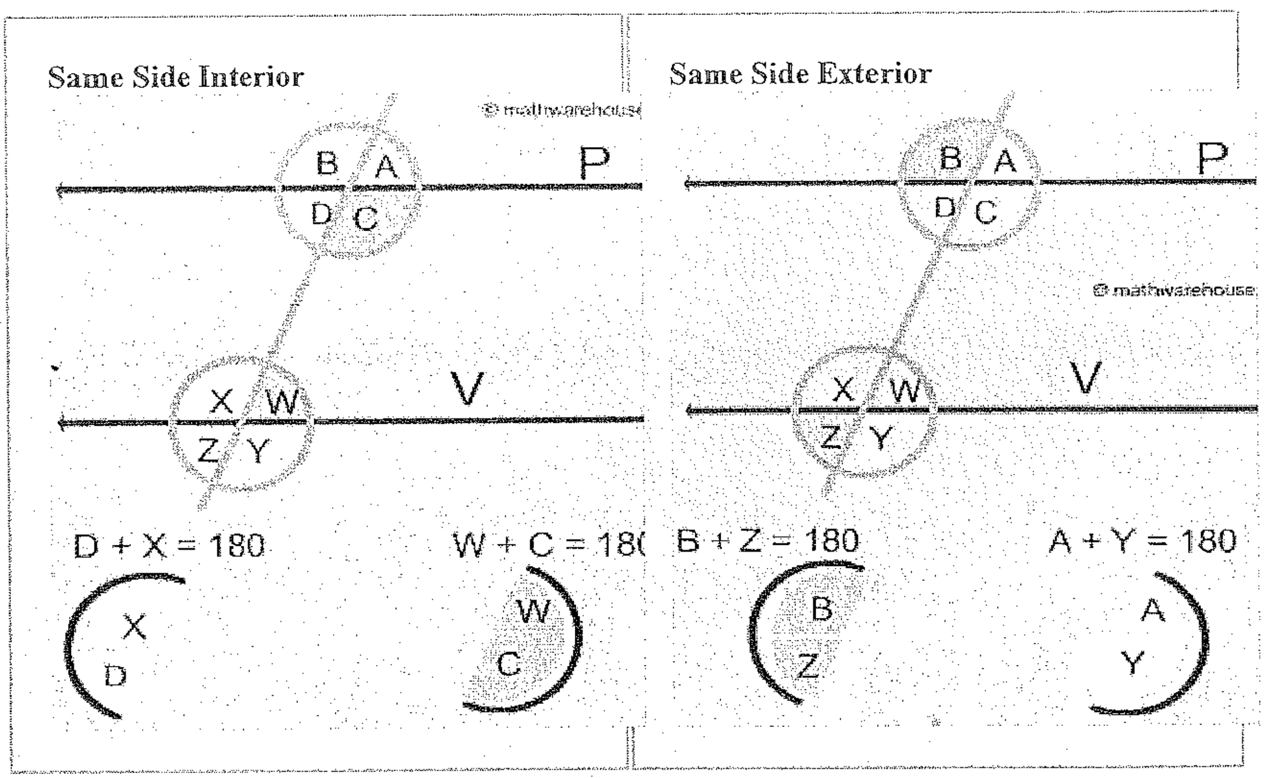
Corresponding Angles

Answer

Load a Similar Problem?

The Supplementary Angle Pairs

There are 2 types of supplementary angles that are formed: Same side interior and same side exterior



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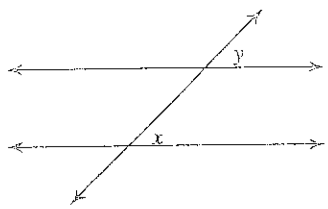
Name___________________________________

Parallel Lines and Transversals

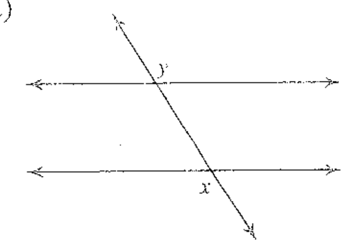
Date__________________ Period____

Identify each pair of angles as corresponding, alternate interior, alternate exterior, or consecutive interior.

1)



2)



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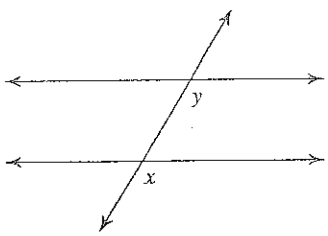

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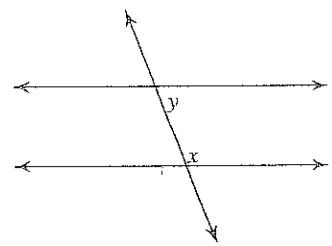

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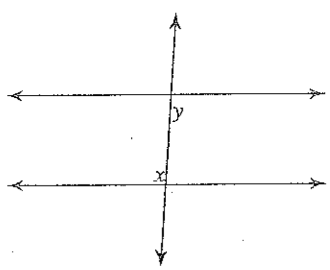

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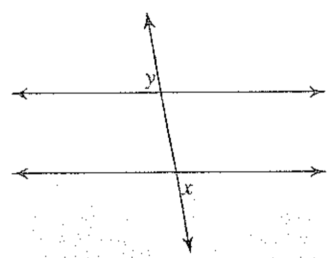

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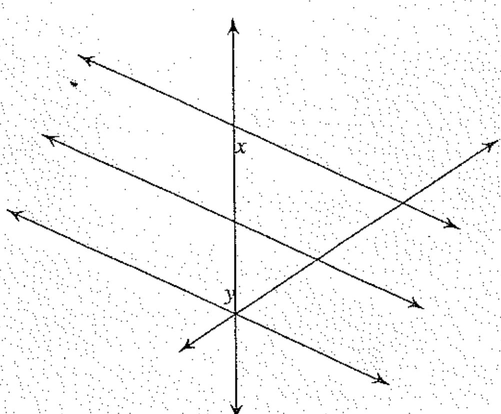

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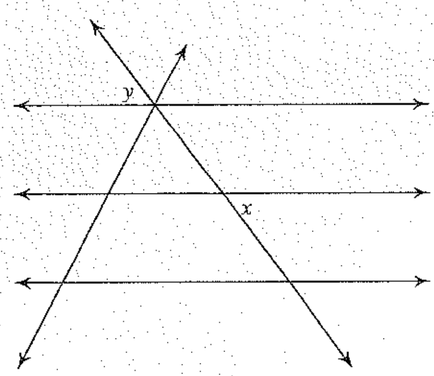

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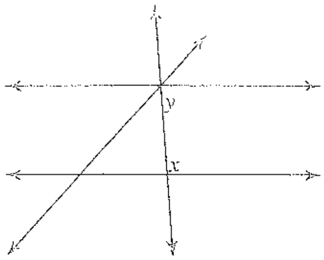

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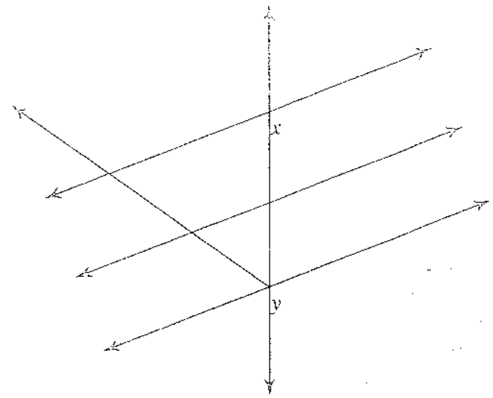
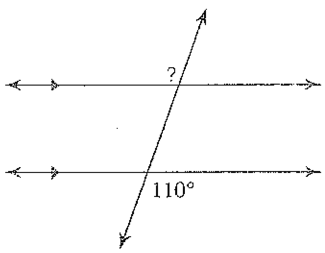
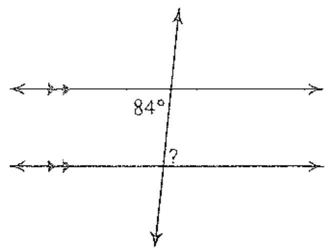


Find the measure of each angle indicated.

11)



12)



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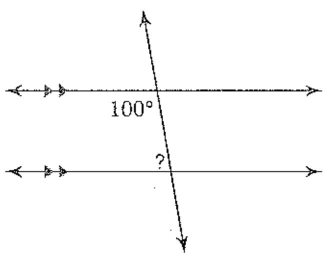

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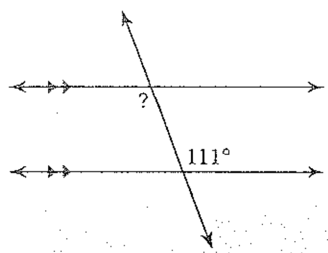

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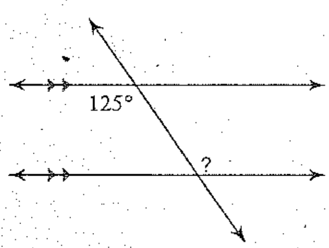

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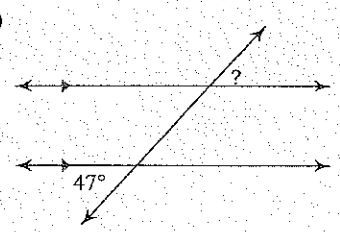

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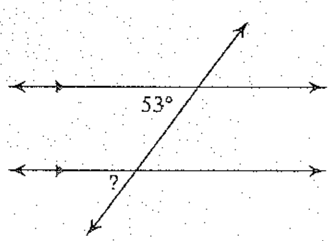
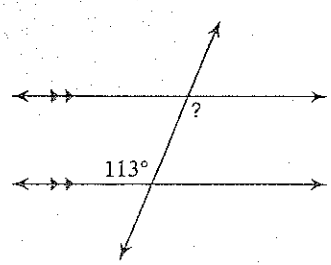
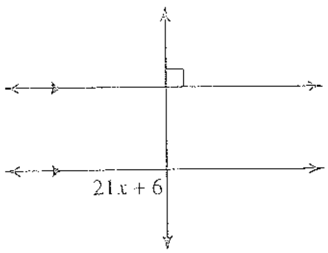


18)

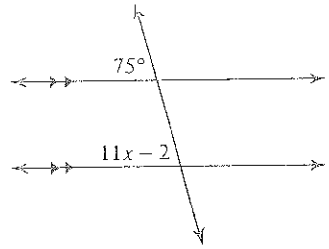


Solve for x.

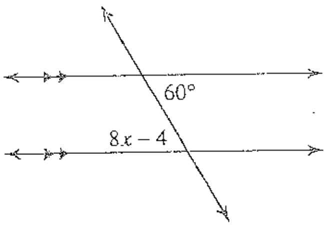
19)



20)



21)



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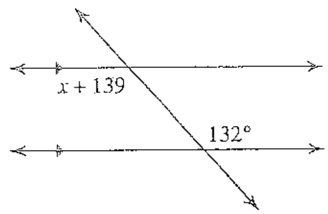
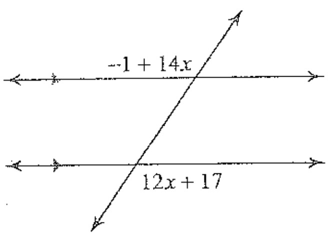
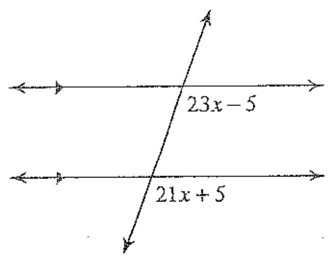


23)

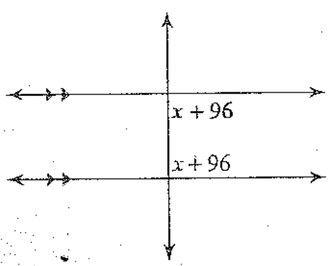


24)



Find the measure of the angle indicated in bold.

25)



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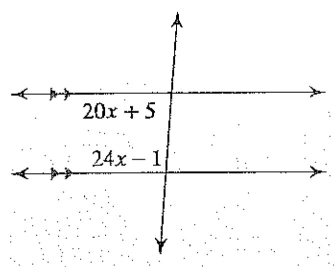
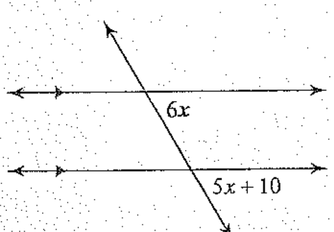
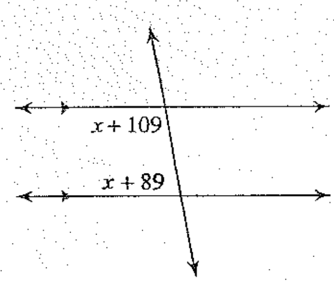


27)



28)

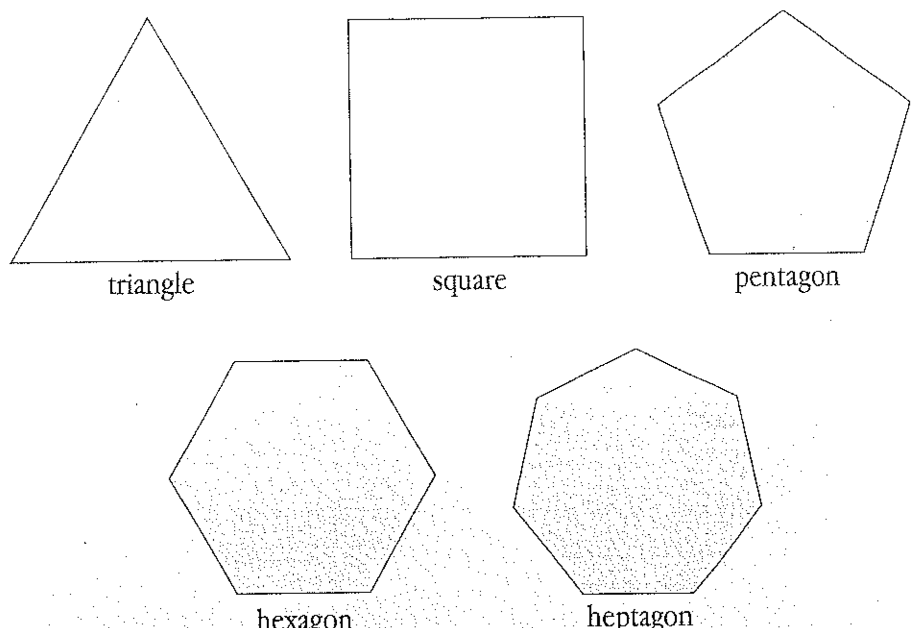


a. Using your cubes, construct a building that fits the clues. Draw a base plan of the building on grid paper.

b. Make a set of building plans for your building.

c. Do you think the building you have constructed is a model of the same building that once existed in Montarek, or do you think there are other possibilities? Explain your reasoning.

8. a. For each of the regular polygons shown below, find the number of lines of symmetry. Organize your data into a table.

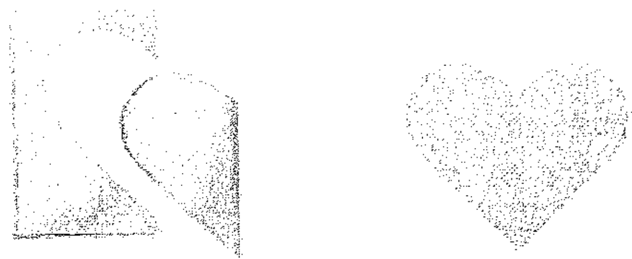


b. Find a pattern in your data that will help you predict how many lines of symmetry a regular polygon with 20 sides will have. Describe your method of predicting.

c. How many lines of symmetry will a regular polygon with 101 sides have?

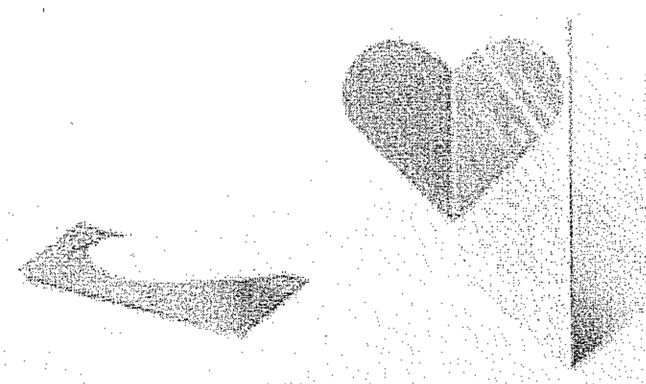
Reflection Symmetry

You have probably made simple heart shapes by folding and cutting paper as shown below.



The resulting heart shape has **reflection symmetry,** which is sometimes called *mirror symmetry* or *line symmetry*. The fold shows the **line of symmetry.** A line of symmetry divides a figure into halves that are mirror images.

If you place a mirror on a line of symmetry, you will see half of the figure reflected in the mirror. The combination of the half-figure and its reflection will have the same size and shape as the original figure. You can use a mirror to check a design for symmetry and to locate the line of symmetry.



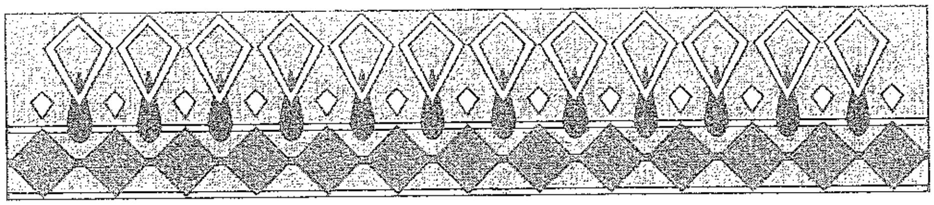
You can also use tracing paper to check for reflection symmetry. Trace the figure and the possible line of symmetry. Then reflect the tracing over the possible line of symmetry. If the reflected tracing fits exactly on the original figure, the figure has reflection symmetry.

What happens to the line of symmetry when you reflect the tracing and match it with the original figure? Does its location change?

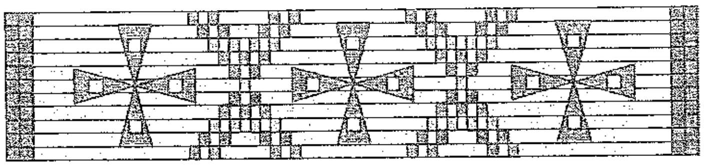
Problem Reflection Symmetry

Use a mirror, tracing paper, or other tools to find all lines of symmetry in each design or figure.

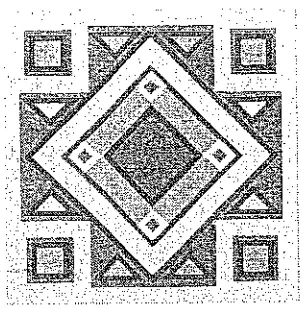
A.



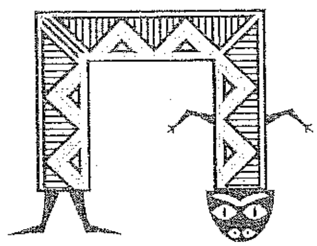
B.



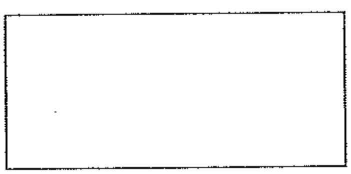
C.



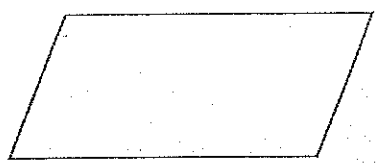
D.



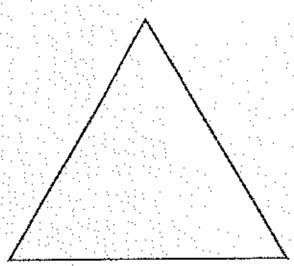
E.



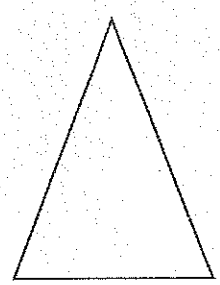
F.



G.



H.

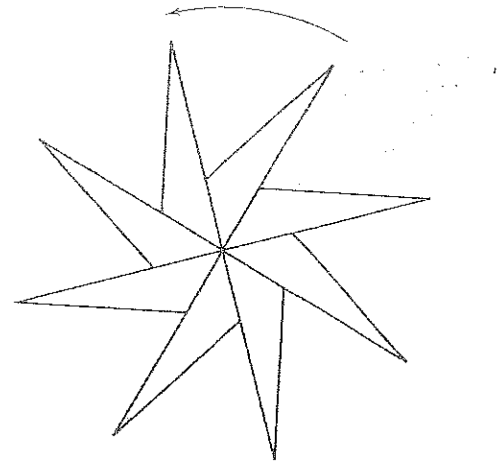


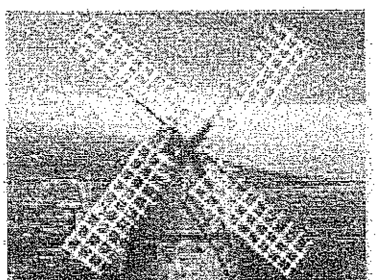
ACE Homework starts on page 15.

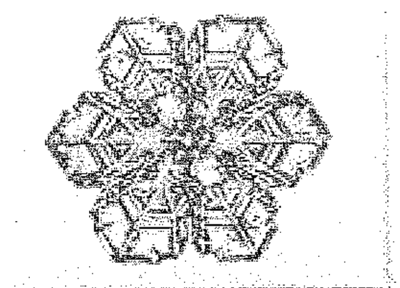
Rotation Symmetry

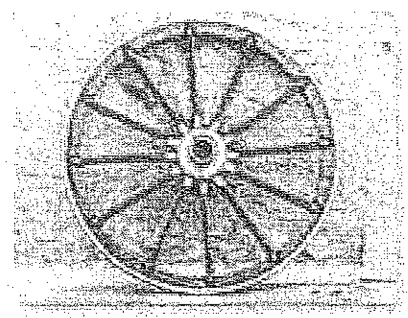
The pinwheel design at the right does not have reflection symmetry. However, it can be turned less than a full turn around its center point in a counterclockwise direction to positions in which it looks the same as it does in its original position. Figures with this property are said to have **rotation symmetry.**

The windmill, snowflake, and wagon wheel pictured below also have rotation symmetry.









Which two of the three objects pictured above also have reflection symmetry?

To describe the rotation symmetry in a figure, you need to specify two things:

- The *center of rotation.* This is the fixed point about which you rotate the figure.
- The *angle of rotation.* This is the *smallest* angle through which you can turn the figure in a counterclockwise direction so that it looks the same as it does in its original position.

There are several rotation angles that move the pinwheel design above to a position where it looks like the original. In this problem, you will consider how these angles are related to the angle of rotation.

Problem Rotation Symmetry

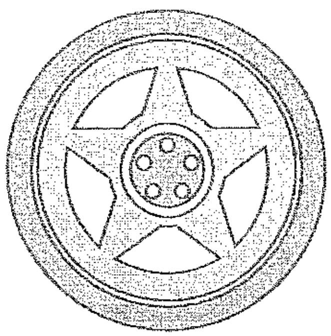
A. List all the turns of less than 360° that will rotate the pinwheel design to a position in which it looks the same as what is pictured. What is the angle of rotation for the pinwheel design?

B. In parts (1)–(3), list all the turns of less than 360° that will rotate the object to a position in which it looks the same as what is pictured. Then give the angle of rotation.

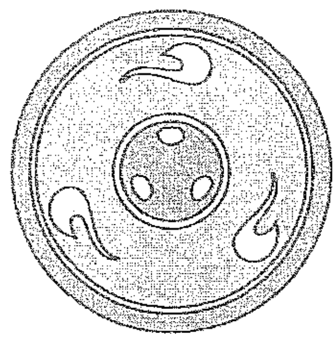
1. the windmill
2. the snowflake
3. the wagon wheel

C. Look at your answers for Questions A and B. For each object or figure, tell how the listed angles are related to the angle of rotation.

D. The hubcaps below have rotation symmetry. Complete parts (1) and (2) for each hubcap.



Hubcap 1



Hubcap 2

1. On a copy of the hubcap, mark the center of rotation. Then, find all the turns of less than 360° that will rotate the hubcap to a position in which it looks the same as what is pictured.
2. Tell whether the hubcap has reflection symmetry. If it does, draw all the lines of symmetry.

E. Draw a hubcap design that has rotation symmetry with a 90° angle of rotation but no reflection symmetry.

F. Draw a hubcap design that has rotation symmetry with a 60° angle of rotation and at least one line of symmetry.

G. Investigate whether rectangles and parallelograms have rotation symmetry. Make sketches. For the shape(s) with rotation symmetry, give the center and angle of rotation.

ACE Homework starts on page 15.

active math online
For: Hubcap Maker
Visit: PHSchool.com
Web Code: apd-5102

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More information

1 Reflection and rotation symmetry

This work will help you

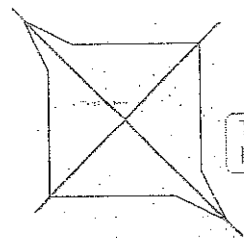
- recognise reflection symmetry and rotation symmetry in a shape
- complete a shape with given reflection or rotation symmetry
- name types of triangle and quadrilateral, and recognise symmetrical and regular polygons

You need sheets F1–1, F1–2, F1–3 and F1–4.

A Reflection symmetry

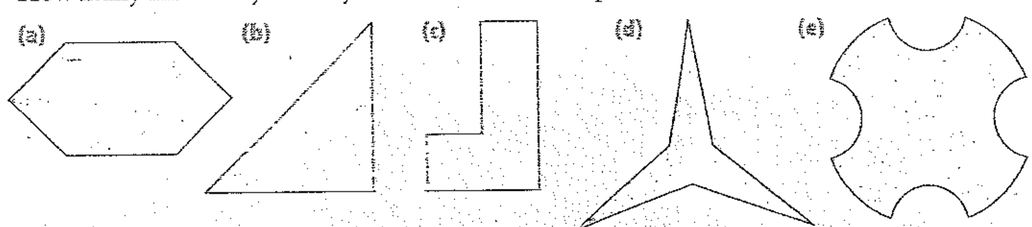
A shape with reflection symmetry has one or more mirror lines that reflect one half of the shape exactly on to the other half.

These mirror lines are called lines of symmetry.



This shape has two lines of symmetry.

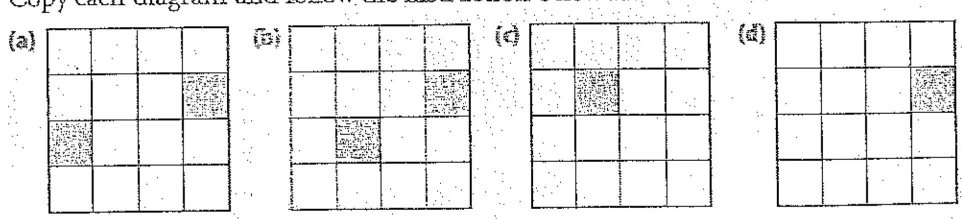
A1 How many lines of symmetry has each of these shapes?



A2 This question is on sheet F1–1.

A3 This question is on sheet F1–2.

A4 Copy each diagram and follow the instruction below it.



(a) Shade two more squares to make a pattern with 2 lines of symmetry.

(b) Shade two more squares to make a pattern with 1 line of symmetry.

(c) Shade three more squares to make a pattern with 4 lines of symmetry.

(d) Shade two more squares to make a pattern with 1 line of symmetry.

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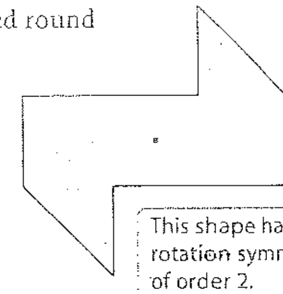
B Rotation symmetry

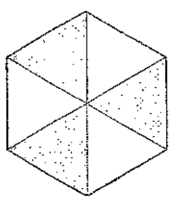
A shape with **rotation** (or rotational) symmetry can be rotated round its centre so that it fits on top of itself more than one way.
The **order** of rotation symmetry is the number of different positions a shape fits on top of itself.

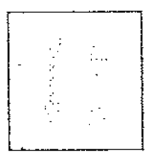


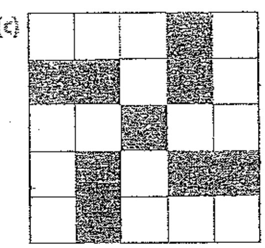
This shape has rotation symmetry of order 2.

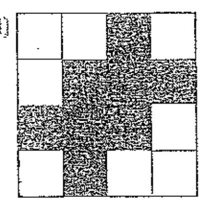
We say that a shape with **no rotation symmetry** has **order 1** as it fits on top of itself in only one way.

B1 What is the order of rotation symmetry of each of these?

(a)


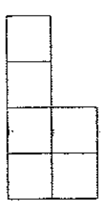
(b)


(c)


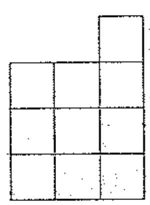
(d)


B2 This question is on sheet F1–3.

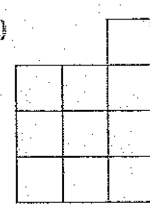
B3 Copy each diagram and complete it as stated.

(a)


Add two more squares so that the final shape has rotation symmetry of order 2.

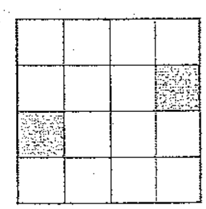
(b)


Add three more squares so that the final shape has rotation symmetry of order 4.

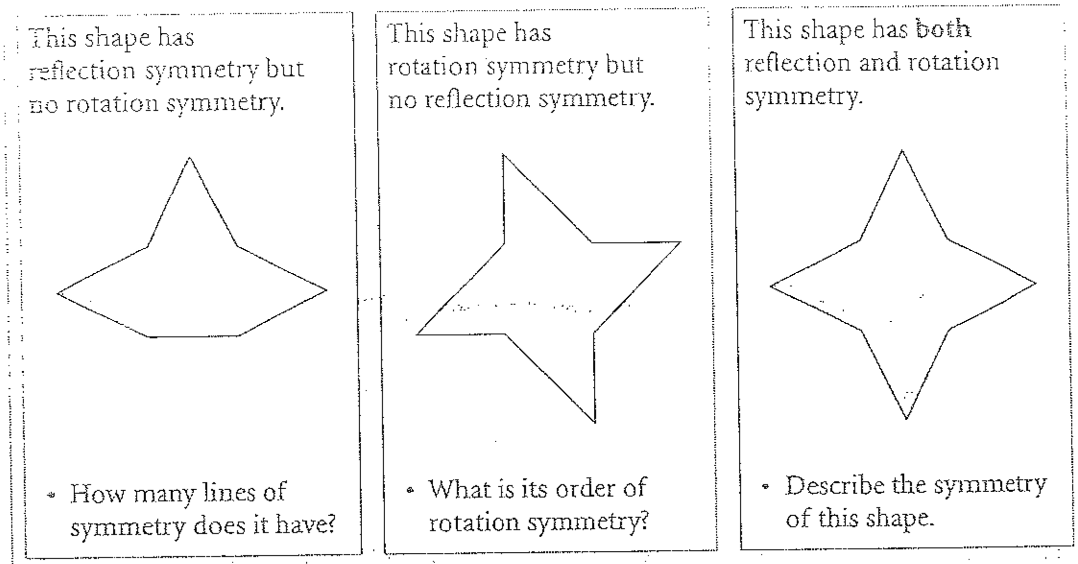
(c)


Add three more squares so that the final shape has no rotation symmetry.

B4 Copy this diagram and shade two more squares so that it has rotation symmetry of order 4.



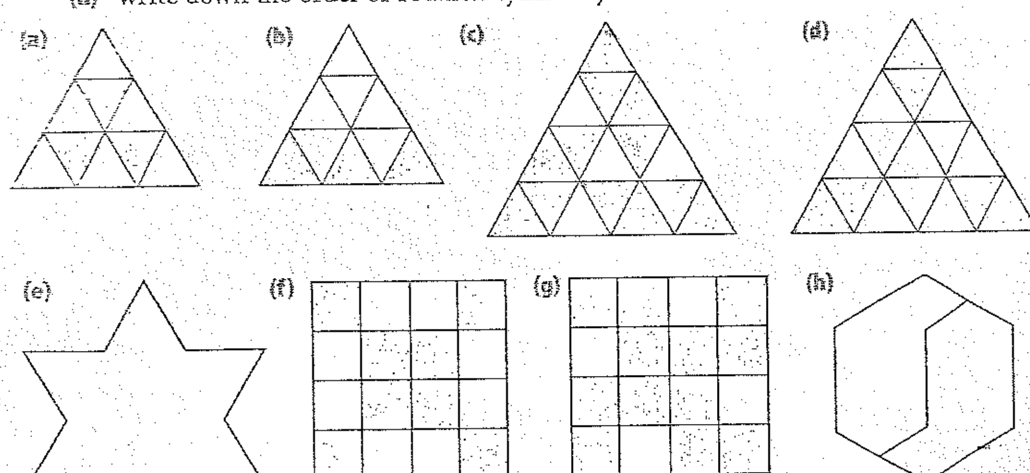
A shape can have reflection symmetry and rotation symmetry



C1 For each design below

(i) write down how many lines of symmetry it has

(ii) write down the order of rotation symmetry



C2 This question is on sheet F1–4.